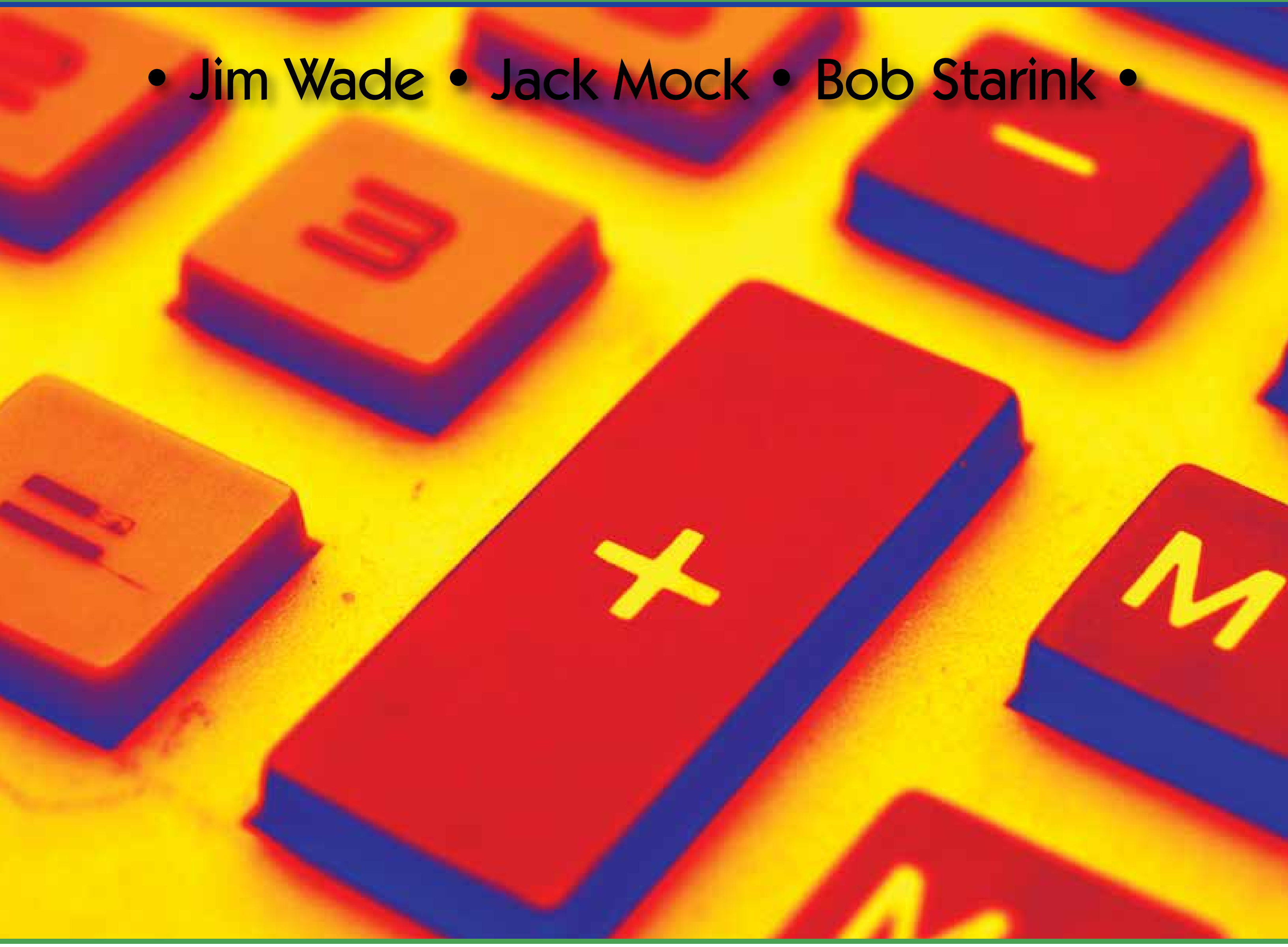
NATIONAL MATHS

YEAR 9

• Jim Wade • Jack Mock • Bob Starink •



S

Science Press

First published 2015

Science Press
Private Bag 7023 Marrickville NSW 1475 Australia
Tel: (02) 9516 1122 Fax: (02) 9550 1915
sales@sciencepress.com.au
www.sciencepress.com.au

Contents

Chapter 1

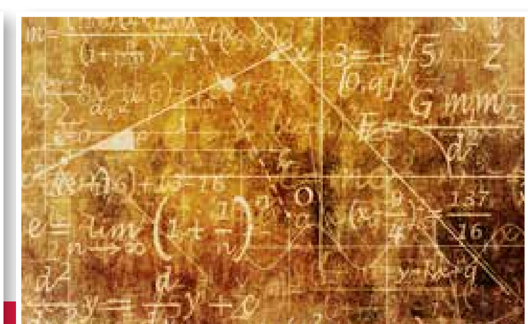
Year 8 Review

KEY SKILLS AND KNOWLEDGE

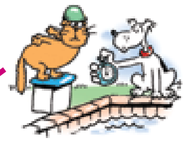
By the end of this chapter you should be able to:

- Carry out the four operations with integers and rational numbers and use index notation with numbers. (1.1)
- Work with real numbers – rational and irrational. (1.2)
- Calculate percentage change, ratio and rates. (1.3)
- Identify and work with congruent figures. (1.4)
- Calculate the probabilities of simple and compound events and use Venn diagrams. (1.5)
- Simplify algebraic expressions including grouping symbols. (1.6)
- Measure length and calculate areas including squares, rectangles and triangles. (1.7)
- Perform calculations with money and GST correctly. (1.8)
- Transform points, lines and shapes by translating, reflecting and rotating. (1.9)
- Solve linear equations by a variety of methods. (1.10)
- Calculate the volume of prisms. (1.11)
- Construct geometrical figures and determine their side, angle and diagonal properties. (1.12)
- Collect, display and summarise statistical data. (1.13)

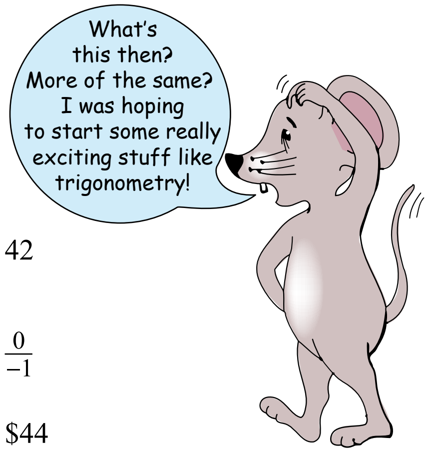




GETTING STARTED



Before undertaking some detailed practice on last year's work, try a quick quiz to see if you can recall some of the basics. If you find difficulty with this set perhaps you had better borrow a year 8 book and have a look over what you have missed.



1. -6×-7 is equal to:
(A) -13 (B) -1 (C) -42 (D) 42

2. $\frac{1}{2} - \frac{1}{3}$ is equal to:
(A) 1 (B) $\frac{5}{6}$ (C) $\frac{1}{6}$ (D) $\frac{0}{-1}$

3. Increase \$40 by 20%. The result is:
(A) \$60 (B) \$48 (C) \$8 (D) \$44

4. Which of these is *not* a test for congruent triangles?
(A) SSS (B) AAA (C) AAS (D) SAS

5. A six-sided die is tossed once. What is the probability of throwing a multiple of 3?
(A) $\frac{1}{6}$ (B) $\frac{1}{2}$ (C) $\frac{1}{3}$ (D) $\frac{2}{3}$

6. Simplify $-2a \times 3ab \times 2b$.
(A) $12ab$ (B) $7ab$ (C) $-12a^2b^2$ (D) $-7a^2b^2$

7. Which quadrilateral has equal diagonals?
(A) A parallelogram. (B) A rectangle. (C) A rhombus. (D) A trapezium.

8. Solve $2x + 3 = 11$.
(A) $x = 4$. (B) $x = 8$. (C) $x = 7$. (D) None of these.

9. The area of a triangle with a base of 10 cm and a height of 4 cm is:
(A) 40 cm^2 (B) 20 cm^2 (C) 60 cm^2 (D) 80 cm^2

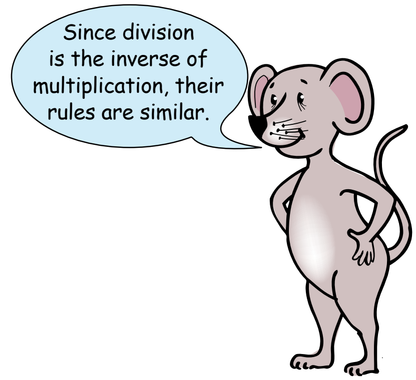
10. The volume of a rectangular box measuring 1.5 m by 30 cm by 20 cm is:
(A) 0.9 m^3 (B) 90 cm^3 (C) 900 000 cm^3 (D) 90 000 cm^3

11. Find the cost of a bike sold for \$60 at 20% profit.
(A) \$66
(B) \$54
(C) \$50
(D) \$48

12. Find the length of the hypotenuse of a right-angled triangle if the other two sides are 8 cm and 15 cm.
(A) 17 cm
(B) 23 cm
(C) 79 cm
(D) 384 cm

1.1 Using the four operations with integers and rational numbers and using index notation with numbers

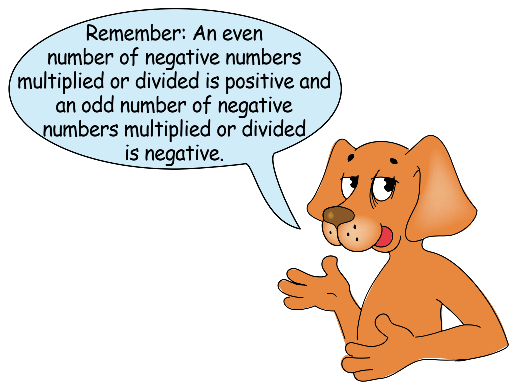
Summary



Positive × positive = positive.	Example: $+3 \times +4 = 12$.
Negative × positive = negative.	Example: $-3 \times +4 = -12$
Positive × negative = negative.	Example: $+3 \times -4 = -12$
Negative × negative = positive.	Example: $-3 \times -4 = 12$.

Easy rule: Two like signs multiply (or divide) to give $+$.

Two unlike signs multiply (or divide) to give $-$.



Example: Evaluate these numerical expressions.

(a) $(-2) \times (-6) \times -3$

(b) $-3 \times -4 \div (-6)$

Solution: (a) $(-2 \times -6) \times -3 = 12 \times (-3) = -36$

(b) $-3 \times -4 \div (-6) = 12 \div (-6) = -2$

Decimals

The rules for multiplying positive and negative decimals and fractions are identical to those for integers.

Example: Calculate:

(a) $0.5 \times (-0.3)$ (b) -0.3×-0.5

Solution: Remove the decimal points, multiply the integers, then insert 2 decimal places.

(a) $5 \times -3 = -15$

$0.5 \times 0.3 = -0.15$

(b) $-3 \times -5 = 15$

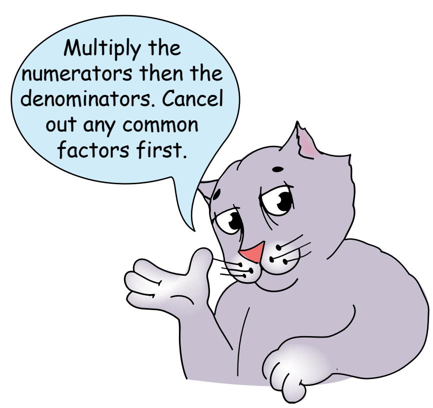
$-0.3 \times (-0.5) = 0.15$

Fractions

Example: Calculate:

(a) $-\frac{2}{3} \times \frac{4}{5}$

(b) $\left(-\frac{3}{4}\right) \times \left(-\frac{2}{5}\right)$



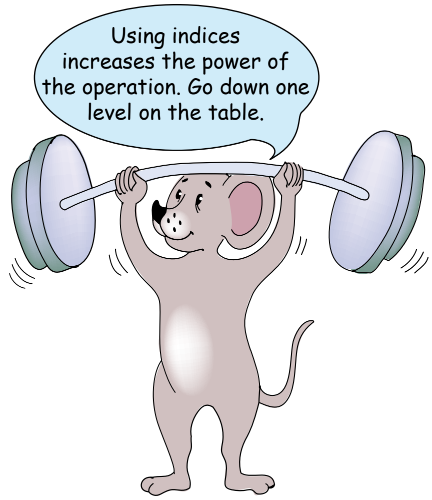
Solution: (a) $-\frac{2}{3} \times \frac{4}{5} = \frac{-2 \times 4}{3 \times 5} = \frac{-8}{15}$

(b) $\left(-\frac{3}{4}\right) \times \left(-\frac{2}{5}\right) = \frac{-3 \times \cancel{-2}}{\underset{2}{\cancel{4}} \times 5} = \frac{3}{10}$

Operations with indices

Recall the order of operations.

	Operation	Inverse operation
Weak	$+$	$-$
Stronger	$\times$	$\div$
Strongest	$(\ldots)^n$	$\sqrt{(\ldots)}$



When evaluating an expression, do the strongest operations first.

Example 1: Evaluate:

(a) $4^3 \times 4^5$ (b) $5^9 \div 5^7$

Solution: (a) $4^3 \times 4^5 = 4^{3+5} = 4^8$
(To multiply, add the indices.)

(b) $5^9 \div 5^7 = 5^{9-7} = 5^2$
(To divide, subtract the indices.)

Example 2: Evaluate:

(a) $(2^4)^2$ (b) $\sqrt[3]{2^{12}}$

Solution: (a) $(2^4)^2 = 2^{4 \times 2} = 2^8$
(To raise one power to another, multiply the indices.)

(b) $\sqrt[3]{2^{12}} = 2^{12 \div 3} = 2^4$
(To find the cube root, divide the index by 3.)

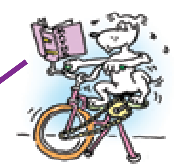
The zero index

Any number divided by itself gives 1.

$2^3 \div 2^3 = 1$ and $2^3 \div 2^3 = 2^{3-3} = 2^0 = 1$

$2^2 \times 3^3$ has no short cut or rule to apply because the powers have different bases.

EXERCISE 1.1



Using the four operations with integers and rational numbers and using index notation with numbers

For the exercises in this set, check every third answer using a calculator.

1. Evaluate:
(a) $3 \times (-2)$ (b) $8 \div (-4)$ (c) -7×5 (d) $-9 \div 3$
(e) $-2 \times (-7)$ (f) $-16 \div (-4)$ (g) $-3 \times (-4) \div 2$ (h) $8 \times 9 \times (-2)$

2. Evaluate the following.
(a) $(-4)^2$ (b) $(-8)^2$ (c) $(-11)^2$ (d) $(-0.3)^2$
(e) $(-3)^3$ (f) $(-4)^3$ (g) $-(-1)^3$ (h) $-10 \times (-5)^3$

3. Multiply or divide these decimals.
(a) -1.8×2 (b) -1.1×-3 (c) 0.4×-1.1 (d) -3.1×-0.3
(e) $0.16 \div (-0.4)$ (f) $(-0.24) \div (-0.6)$ (g) $(-0.4)^2$ (h) $(-0.2)^3$

4. Multiply or divide these fractions.

(a) $\frac{1}{4} \times -\frac{1}{3}$ (b) $-\frac{3}{4} \times -\frac{2}{5}$ (c) $\left(-\frac{4}{5}\right)^2$ (d) $-\left(\frac{-3}{4}\right)^3$

(e) $-\frac{1}{2} \times -\frac{2}{5} \times -\frac{5}{6}$ (f) $-1\frac{1}{2} \times 3\frac{1}{2}$ (g) $-2\frac{1}{4} \div -1\frac{1}{3}$ (h) $-1\frac{1}{2} \times -2\frac{2}{3} \div -3\frac{3}{4}$

5. Use your calculator to obtain:

(a) $\sqrt{36}$ (b) $\sqrt{324}$ (c) x if $x^2 = 256$. (d) y if $y^2 = 81$.

6. Two more than –8 is added to the product of 4 and 6 less than 2. What is the result?

7. Two numbers have a product of 30 and a sum of −11. Find the two numbers.

8. Expand these powers and write each of them as an integer.

(a) 2^3 (b) 3^4 (c) $(-2)^5$

(d) $(-5)^3$ (e) $(-2)^6$ (f) $(-10)^7$

9. Write each of these powers in index form using the indicated base.

(a) 16 (base 2). (b) 36 (base 6). (c) −8 (base −2). (d) −1000 (base −10).

10. Simplify these expressions, taking care to apply the correct order of operations.

(a) $4^2 \times 3 + 2$ (b) $4 + 5 \times 2^3$ (c) $5 - (2 \times 3)^2$ (d) $(6 - 2) \times 3^2$ (e) $3^2 \times 5 - 4 \times 2^3$

11. Write the answer to these products and quotients, leaving your answer in index form.

(a) $5^2 \times 5^8$ (b) $3^{49} \times 3$ (c) $8^0 \times 8^2$ (d) $2^7 \div 2^3$ (e) $17^5 \div 17^3$

12. Expand these brackets, leaving your answer in index form.

(a) $(2^3)^5$ (b) $(3^2)^4$ (c) $(10^6)^2$ (d) $(5^{99})^2$ (e) $(3^3)^3$

13. Find these roots, leaving your answer in index form.

(a) $\sqrt{2^{16}}$ (b) $\sqrt{16^4}$ (c) $\sqrt[3]{2^{12}}$ (d) $\sqrt{4^{100}}$ (e) $\sqrt[5]{10^{20}}$

14. Simplify these index calculations.

(a) $(5^3 \times 5^{24})^0$ (b) $(2^0 \times 2^{99})^2$ (c) $\sqrt[3]{7^2 \times 7^{16}}$ (d) $10^{15} \div 10^3$

15. Use this table of the powers of 2 to perform these calculations using a shortcut method with indices.

2^0	2^1	2^2	2^3	2^4	2^5	2^6	2^7	2^8	2^9	2^{10}	2^{11}	2^{12}
1	2	4	8	16	32	64	128	256	512	1024	2048	4096

(a) 128×16 (b) 32^2 (c) $2048 \div 256$

(d) 16^3 (e) $\sqrt[3]{4096}$

1.2 Working with real numbers – rational and irrational

Fractions with denominators divisible only by powers of 5 and 2 are terminating decimals. All other fractions have decimals that recur.

To convert a **terminating decimal** to a fraction, it is only necessary to place the digits over a power of 10 with the same number of zeros as decimal places.

Example: Convert 0.375 to a fraction in its lowest terms.

Solution: $0.375 = \frac{375}{1000} = \frac{15}{40} = \frac{3}{8}$

The conversion of **recurring decimals** follows a simple pattern. Single digit recurring decimals are expressed as 9ths, double digit recurring decimals as 99ths and so on.

Example: Convert these recurring decimals to fractions in their lowest terms.

(a) $0.\dot{4}$ (b) $0.\dot{6}\dot{5}$

Solution: (a) $0.\dot{4} = \frac{4}{9}$ (b) $0.\dot{6}\dot{5} = \frac{65}{99} = \frac{13}{33}$

To convert a partially recurring decimal to a fraction, separate the non-terminating and terminating decimal parts, convert each to a fraction and add.

Example: Convert $0.8\dot{3}$ to a fraction.

Solution: $0.8\dot{3} = 0.8 + 0.0\dot{3} = \frac{8}{10} + \frac{0.\dot{3}}{10} = \frac{4}{5} + \frac{^3\!/_9}{10} = \frac{4}{5} + \frac{1}{30} = \frac{25}{30} = \frac{5}{6}$

All terminating and recurring decimals can be written as fractions and are called **rational numbers**.

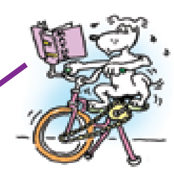
Non-terminating, non-recurring decimals cannot be written as fractions and are called **irrational numbers**.

Square roots of non-square numbers are irrational while square roots of square numbers are rational.

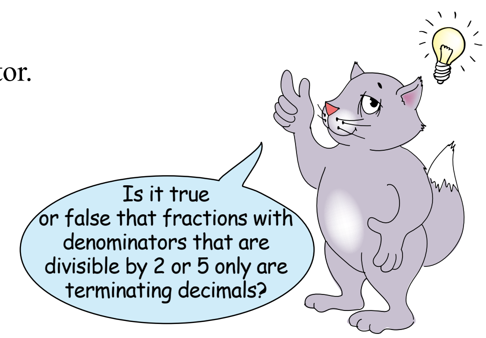
Example 1: $\sqrt{1980} = \sqrt{2^2 \times 3^2 \times 11 \times 5}$ Irrational as 11 and 5 are odd powers.

Example 2: $\sqrt{1764} = \sqrt{2^2 \times 3^2 \times 7^2}$ Rational as all of the factors are even powers.

EXERCISE 1.2



Working with real numbers – rational and irrational



1. Convert each of these fractions to decimals by dividing the numerator by the denominator.

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$

(c) $\frac{1}{5}$ (d) $\frac{3}{8}$

(e) $\frac{1}{3}$ (f) $\frac{1}{7}$

(g) $\frac{2}{9}$ (h) $\frac{5}{6}$

2. Convert these terminating decimals to fractions cancelled down to their lowest terms.

(a) 0.6 (b) 0.24 (c) 0.675 (d) 0.0875 (e) 0.0025

3. Convert these recurring decimals to fractions cancelled down to their lowest terms.

(a) $0.\dot{8}$ (b) $0.\dot{5}\dot{3}$ (c) $0.\dot{5}\dot{4}$ (d) $0.\dot{5}4\dot{1}$ (e) $0.\dot{1}00\dot{2}$

4. Convert these partially recurring decimals to fractions in their lowest terms.

(a) $0.5\dot{3}$ (b) $0.41\dot{6}$

5. Using your calculator, write down the following recurring decimals and compare the results for each one. What pattern can you recognise?

(a) $\frac{1}{11}$ (b) $\frac{2}{11}$ (c) $\frac{3}{11}$ (d) $\frac{4}{11}$

6. Again using your calculator write down the following recurring decimals.

(a) $\frac{1}{13}$ (b) $\frac{2}{13}$ (c) $\frac{3}{13}$ (d) $\frac{4}{13}$

7. (a) Using your results from Questions 5 and 6, write down each set of repeating digits as an integer.

(b) Use your calculator to see if this number is divisible by 9.

(c) Complete this statement: The repeating digits of any fraction with a prime denominator greater than 5 form a number that is divisible by ....

8. To calculate an approximation for the *irrational number* $\sqrt{2}$, you will need a calculator.

(a) Make a first approximation for $\sqrt{2} \approx 1$. (Note the sign '$\approx$' means 'approximately equal to'.)

(b) Now square the approximation $(1)^2 = 1$ and find the difference from 2. (Difference $2 - 1 = 1$.)

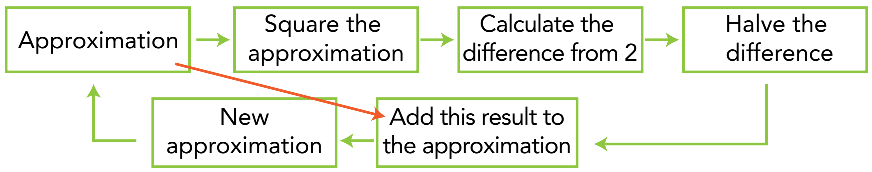
(c) Halve the difference and add this to the first approximation. $1 + 0.5 = 1.5$.

(d) Second approximation = 1.5. Squaring = 2.25 and finding the difference $2 - 2.25 = -0.25$.

(e) Halve the difference (-0.125) and add it to 2nd approximation $1.5 + (-0.125) = 1.375$.

(f) Using your calculator, perform 2 more steps of this algorithm. Compare your answer to the answer provided by your calculator for $\sqrt{2}$. Is it correct to 3 significant figures?

Here is a flow chart to do this.



9. By reducing these numbers to powers of their prime factors, decide if the square roots are rational or irrational numbers. Check your answers with a calculator.

(a) $\sqrt{12}$ (b) $\sqrt{225}$ (c) $\sqrt{576}$ (d) $\sqrt{588}$

10. For each of these numbers state whether they are rational or irrational.

(a) $7\sqrt{3}$ (b) $100\sqrt{5}$ (c) $5\sqrt{100}$ (d) $9\sqrt{2}$ (e) $2\sqrt{9}$

(f) $17\sqrt{5}$ (g) $1 + \sqrt{2}$ (h) $1 + \sqrt{4}$ (i) $25 - \sqrt{5}$ (j) $5 - \sqrt{25}$

11. Complete this statement: The irrational number π is defined as $\frac{\text{circumference}}{\ldots}$.

12. Which one of these is *not* an approximation for π?

(A) 3.142 (B) $\frac{22}{7}$ (C) $3\frac{1}{7}$ (D) 3.28

1.3 Calculating percentage change, ratio and rates

To find a percentage of a quantity, write the percentage as a decimal or fraction and then multiply by the quantity.

Example: (a) Find 15% of \$850 using decimals.

(b) Find 35% of \$720 using fractions.

Solution: (a) $(15 \div 100) \times 850 = 0.15 \times 850 = \$127.50$

(b) $\frac{\cancel{35}^{7}}{\cancel{100}} \times \cancel{720}^{36} = \$252$

Note: When using a calculator it is usually easier to change the percentage to a decimal rather than changing to a fraction to solve percentage problems.

One quantity as a percentage of another

Example: What percentage of 2 L is 800 mL?

Solution: Write 800 mL as a fraction of 2000 mL.

$\frac{\cancel{800}}{\cancel{2000}} \times \frac{\cancel{100}^{5}}{1}\% = 40\%$.

Percentage composition

Percentages are sometimes used to give the composition of mixtures.

Example 1: Chummy dog food contains 400 g protein, 40 g fat, 5 g fibre and 5 g salt.

Find the percentage composition of the ingredients.

Solution: Total = 400 + 40 + 5 + 5 = 450 g.

Protein = $\frac{400}{450} \times 100\% = 89\%$. Fat = $\frac{40}{450} \times 100\% = 9\%$.

Fibre = $\frac{5}{400} \times 100\% = 1\%$. Salt = 1%.

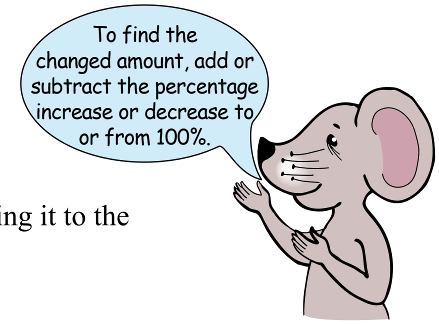
Example 2: Kirra earns \$750 per week.

Calculate her new wage if she receives a pay rise of $3\frac{1}{2}\%$.

Solution: Add 3.5% to the original 100% = 103.5%.

1.035 × 750 = \$776.25

Note: This is a much more efficient method than calculating 3.5% and adding it to the original wage.



Dividing a quantity in a given ratio

Example: The ratios of the blood types O, A, B and AB in the population are approximately 10 : 10 : 4 : 1.

How many units of each type should a hospital requiring 2000 units of blood receive?

	O	A	B	AB	Total
Ratio	10	10	4	1	25
Blood units	x	y	z	w	2000

Solution: $\frac{x}{10} = \frac{y}{10} = \frac{z}{4} = \frac{w}{1} = \frac{2000}{25}$

$x = y = \frac{20\,000}{25} = 800$ units (O, A). $z = \frac{8000}{25} = 320$ units (B). $w = 80$ units (AB).

Calculating rates

Example: 600 metres is travelled in 3 minutes.

What is this speed in km/h?

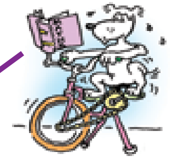
Solution: Distance : time

= 600 metres : 3 minutes

= 12 000 metres : 60 minutes
(× 20 to obtain 1 hour using the unitary method.)

= 12 km/h.

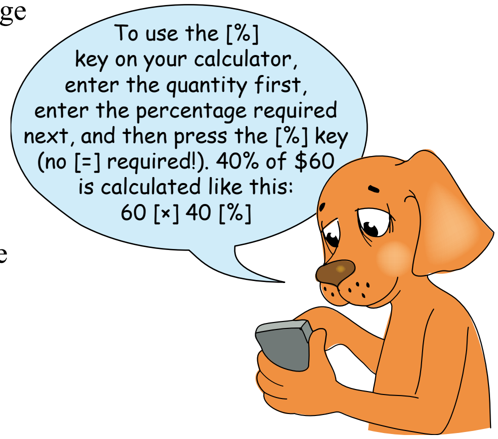
EXERCISE 1.3



Calculating percentage change, ratio and rates

1. Find the percentage of the given quantity by changing the percentage to a decimal.
(a) 15% of \$250. (b) 45% of 700 mL. (c) 84% of 250 cm. (d) 120% of 3.6 L.

2. Find the percentage of the quantity by changing the percentage to a fraction.
(a) 40% of 500 kg.
(b) 15% of \$45.
(c) 85% of 1500 g.
(d) 70% of 30 min.

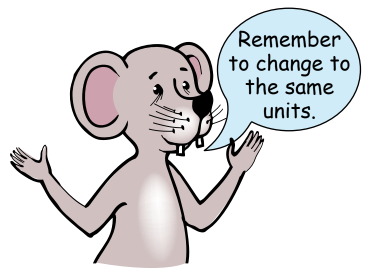


3. Use the percentage key (if your calculator has one) or change to a decimal before using your calculator to find these quantities.
(a) 30% of 1.4 kg. (b) 18% of 7.5 m.
(c) 24% of 15 km. (d) 45% of 0.6 t.

4. Anna earns \$850 per week. She is given a pay rise of 3%. What is the amount of the increase?

5. Which is the greater sum of money and by how much: $37\frac{1}{2}$% of \$600 or $8\frac{1}{3}$% of \$2580?

6. Express the first quantity as a percentage of the second.
(a) 300 cm, 750 cm (b) 18 min, 90 min
(c) 15 kg, 40 kg (d) 75 t, 1000 t
(e) \$18, \$72 (f) 85 cm, 1 m
(g) 400 kg, 2 t (h) 720 mL, 2 L
(i) 45 min, 4 h (j) 6000 m^2, 3 ha



7. In the class there were 16 boys and 9 girls. What is the percentage composition of the class (that is, what percentage are boys and what percentage are girls)?

8. Nickel silver contains 2 parts copper, 1 part zinc and 1 part nickel. What is its percentage composition?

9. An artist mixed the following paint pigments to obtain the required colour of paint: 25 g white, 50 g yellow and 125 g green. Find the percentage of each pigment used.

10. Calculate these percentage changes.
(a) Increase \$45 by 20%. (b) Increase 8 kg by 40%. (c) Increase 2.4 m by 15%.
(d) Decrease \$600 by 30%. (e) Decrease 40 kg by 40%. (f) Decrease 6.4 L by 60%.

11. Australia exports live sheep to the Middle East but there is a fatality rate of about 4% on a normal run. If a container ship carries 8500 sheep, how many sheep survive the trip?

12. If 50 L was lost through a leak and this represented 5% of a tank, find the total capacity of the tank.

13. Mrs Jones received a 30% discount off the purchase price of a television set. If she saved \$240, find the marked price of the TV.

14. Find 20% of 40% of \$320. Is this the same as finding 40% of 20% of \$320?

15. An investor lost 20% on a falling stock market. By what percentage must the market now increase, in order to restore the investment to the original value?

16. Use the unitary method to find how much cement, sand and gravel must be mixed in the ratio 1 : 2 : 3 in order to produce 16.8 cubic metres of concrete mix?

17. In an exam there are 10 questions in section A and 20 questions in section B. The time allowed is 60 minutes. Divide the time for section A and B in the ratio of the number of questions.

18. The 3 sides of a triangle are in the ratio 3 : 4 : 5. If the perimeter is 2640 mm, calculate the lengths of the sides.

19. The golden ratio is 1.618 : 1. This means that any golden rectangle will have the ratio of its length to its width equal to the golden ratio.

(a) Find the length of the sides of a golden rectangle that has a perimeter of 60 cm.

(b) Find the perimeter of a golden rectangle with a short side of 5 cm.

20. Calculate these speeds and express them in the indicated units.

(a) A car travels 80 km in $1\frac{1}{2}$ hour. (km/h.)

(b) A bushwalker walked 7 km in $3\frac{1}{2}$ hours. (km/h.)

(c) A snail crawls 40 cm in 5 minutes. (cm/min.)

(d) A plant grew 18 mm in 24 hours. (mm/h.)

21. A cricketer hit 65 runs off 50 balls.

(a) What is his run rate in runs/ball?

(b) Calculate his strike rate which is expressed in runs per 100 balls.

22. Global warming is currently estimated to be proceeding at around 0.5º per hundred years. This rate is expected to treble next century. If an increase in global temperature of 4.5º spells disaster for the planet, how long have we got to fix it?

23. \$100 AUD can be exchanged for US\$95 (American).

(a) What is the currency exchange rate expressed as A\$ per US\$?

(b) What is US\$25 worth in Australian currency?

24. A canoeist can paddle at 8 km/h in flat water. Today he is paddling in a river that flows at 5 km/h.

(a) What is his speed of paddling downstream?

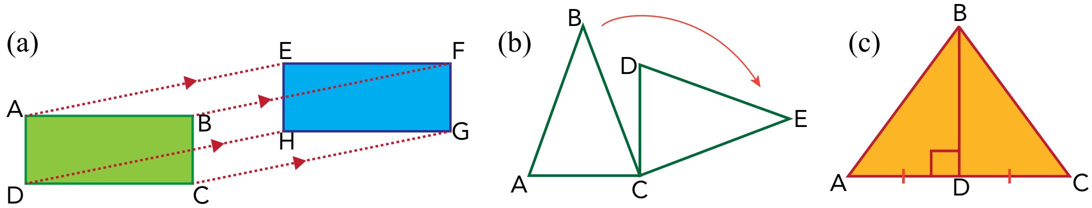
(b) What is his speed of paddling upstream?

(c) He paddles 12 km downstream and returns. What is his average speed for the round trip?

1.4 Congruence

Identifying congruent shapes

Example: Are the two figures congruent?

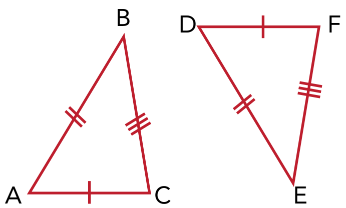


Solution: If a figure can be superimposed on another, say by translation, rotation or reflection, or any combination of these, then they are identical. That is, they are congruent figures.

(a) ABCD ≡ EFGH (b) $\Delta ABC \equiv \Delta DEC$ (c) $\Delta ABD \equiv \Delta CBD$

Tests for congruent triangles

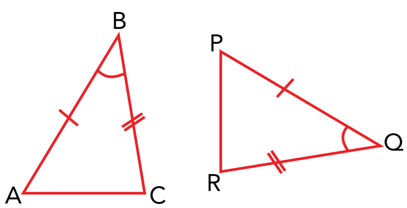
The three sides test (SSS)



If three sides in one triangle are respectively equal to three sides in another triangle then the triangles are congruent (SSS).

$\Delta ABC \equiv \Delta DEF$ (SSS).

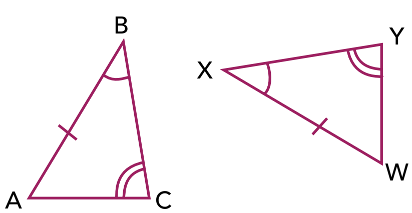
Two sides and the included angle test (SAS)



If two sides and the *included* angle of one triangle are respectively equal to two sides and the *included* angle in another triangle then the triangles are congruent (SAS).

$\Delta ABC \equiv \Delta PQR$ (SAS).

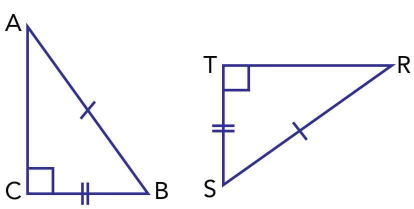
Two angles and a corresponding side (AAS)



If two angles and one side in one triangle are respectively equal to two angles and the *corresponding* side in another triangle then the triangles are congruent (AAS).

$\Delta ABC \equiv \Delta WXY$ (AAS).

Right angle, hypotenuse and one side (RHS)



If the hypotenuse and one other side in one right-angled triangle are respectively equal to the hypotenuse and one other side in another right-angled triangle then the triangles are congruent (RHS).

$\Delta ABC \equiv \Delta RST$ (RHS).

Congruence proofs

Example: Prove $\Delta APQ \equiv \Delta BPQ$ given that ΔAPB is an isosceles triangle with $AP = BP$.

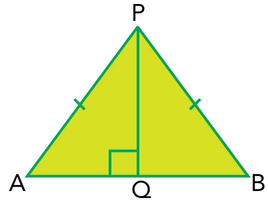
Solution: In the Δs APQ and BPQ

$AP = BP$ (equal sides, isosceles Δ)

$\angle AQP = \angle BQP = 90°$ (given)

PQ is a common side

$\therefore \Delta APQ \equiv \Delta BPQ$ (RHS)

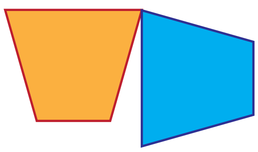


EXERCISE 1.4

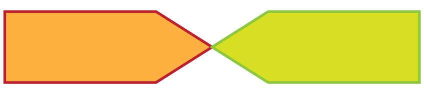
Congruence

1. Which of the following diagrams show two congruent figures?

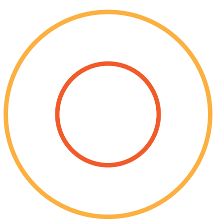
(a)



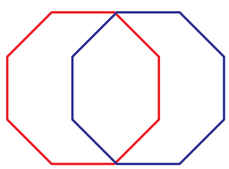
(b)



(c)

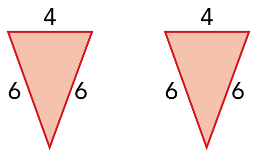


(d)

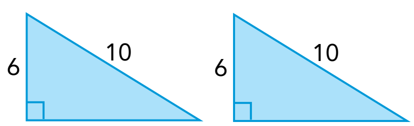


2. Which congruence test (SSS, SAS, AAS, RHS) would you use to prove the following pairs of triangles congruent?

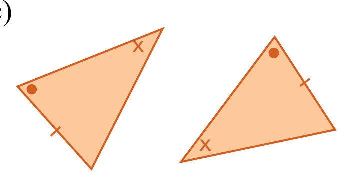
(a)



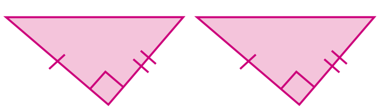
(b)



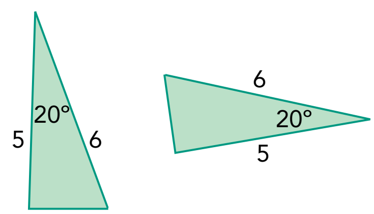
(c)



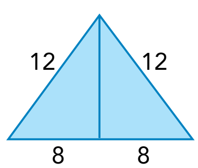
(d)



(e)

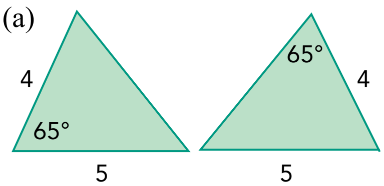


(f)

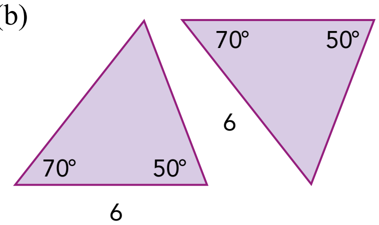


3. Are these two triangles congruent? Explain.

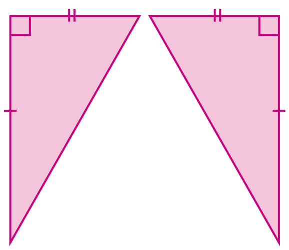
(a)



(b)

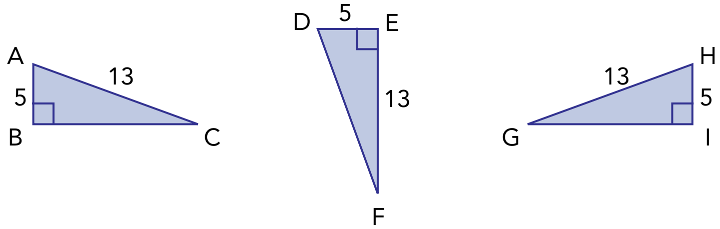


(c)

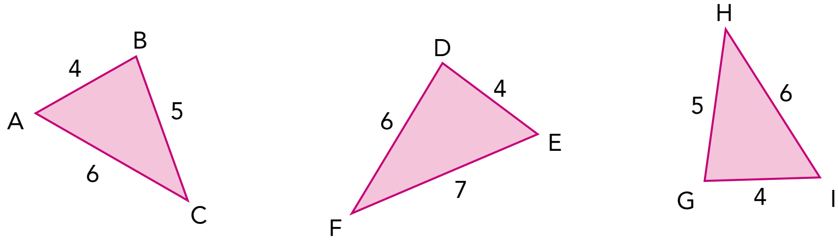


4. Which pair of triangles are congruent to each other? Give reasons.

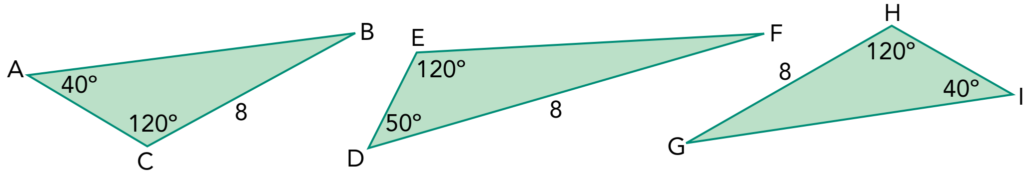
(a)



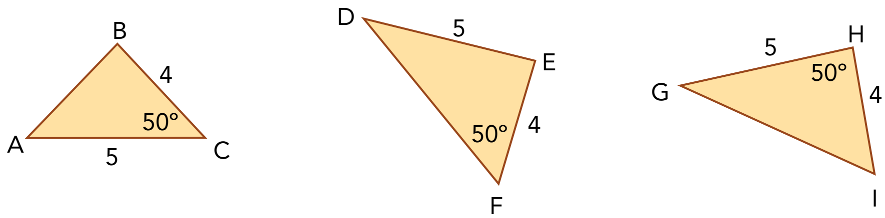
(b)



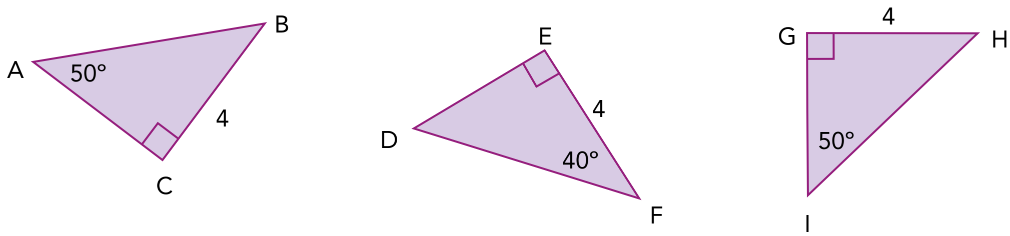
(c)



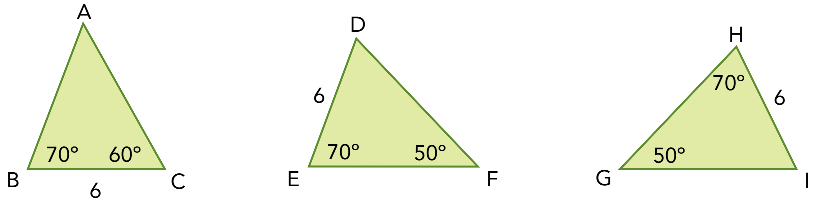
(d)



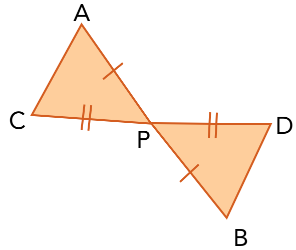
(e)



(f)



5. Given that AP = BP and CP = DP, prove that ΔACP ≡ ΔBDP and hence show that AC = BD.



1.5 Chance and compound events

Simple probability

Example: A fair, six-sided die is rolled. What is the probability of obtaining the following outcomes?

(a) The number 4.
(b) A number less than 4.
(c) An even number.
(d) A number less than 8.

Solution: (a) $\text{Probability} = \dfrac{\text{number of ways event happens}}{\text{number of points in sample space}} = \dfrac{1}{6}$

(b) $P = \dfrac{3}{6} = \dfrac{1}{2}$ (c) $P = \dfrac{3}{6} = \dfrac{1}{2}$ (d) $P = 1$ (Must happen.)

The probability of the complementary event

Example: Find the probability that a card drawn at random from a well-shuffled pack of playing cards is not a king.

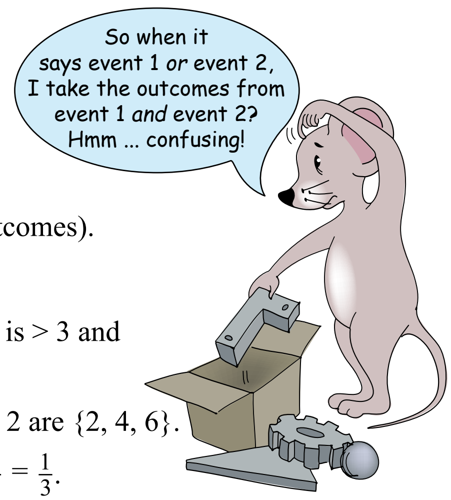
Solution: $P(\text{not king}) = 1 - P(\text{king}) = 1 - \dfrac{4}{52} = 1 - \dfrac{1}{13} = \dfrac{12}{13}$

Expected number of successes

If 20 people toss a coin, the probability of a head is $\frac{1}{2}$ in each case.

We expect that on half of the occasions when a coin is tossed, it will come down heads.

Therefore we will expect $\frac{1}{2} \times 20 = 10$ heads.



Compound events

Example 1: A die is thrown. What is the probability of an even number or a one?

Solution: The events covered include 2, 4, 6 *and* 1 (4 different outcomes).

$P = \dfrac{4}{6} = \dfrac{2}{3}$

Example 2: A die is thrown. What is the probability that the number is > 3 and divisible by 2?

Solution: The numbers > 3 are {4, 5, 6}; the numbers divisible by 2 are {2, 4, 6}.

The outcomes that fit both descriptions are {4, 6}. $P = \dfrac{2}{6} = \dfrac{1}{3}$.

At least

Sometimes it is easier to look at what is not happening than what is actually happening.

Suppose we want to pick a number from 1 to 100 that is at least 10. That is, it is not from 1 to 9.

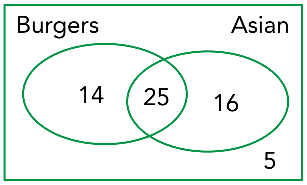
$P = 1 - P(1 \text{ to } 9) = 1 - \dfrac{9}{100} = \dfrac{91}{100}$.

At most

Similarly, if we put 10 discs numbered 1 to 10 in a bag and draw one out, what is the probability it is at most 9? If it is at most 9, then it is anything other that 10.

$P(\text{not } 10) = 1 - P(10) = 1 - \dfrac{1}{10} = \dfrac{9}{10}$.

Using Venn diagrams to calculate probability



Example: A group of 60 people are surveyed to find their taste in food. There are 39 that like burgers and 41 like Asian food. 5 people like neither. The diagram shows that 25 people like both burgers and Asian food.

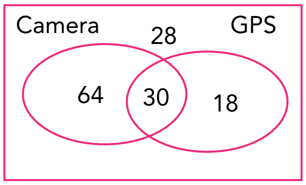
If one person is selected at random, find the probability that they like:

(a) Only burgers. (b) Asian food.
(c) Both burgers and Asian food. (d) Only one sort of food.
(e) Either burgers or Asian. (f) Neither.

Solution: (a) $P = \frac{14}{60} = \frac{7}{30}$ (b) $P = \frac{41}{60}$ (c) $P = \frac{25}{60} = \frac{5}{12}$
(d) $P = \frac{30}{60} = \frac{1}{2}$ (e) $P = \frac{55}{60} = \frac{11}{12}$ (f) $P = \frac{5}{60} = \frac{1}{12}$

Two-way tables (another way of displaying information)

Example: In a technology survey people are asked if they own a camera or not and if they own a GPS device or not. The numbers in each category can be displayed either in a Venn diagram or a two-way table.

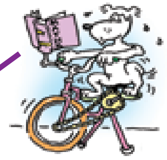


	Camera owner	Non-camera owner	Totals
GPS owner	30	18	48
Non-GPS	64	28	92
Totals	94	46	140

(a) If one person is chosen, what is the probability they do not own a camera but do have a GPS?
(b) If a camera owner is selected at random, what is the probability they also own a GPS?
(c) If a non-camera owner is selected, what is the probability that they also do not own a GPS?
(d) If a GPS owner is selected, what is the probability that they do not have a camera?

Solution: (a) $P = \frac{18}{140} = \frac{9}{70}$ (b) $P = \frac{30}{94} = \frac{15}{47}$ (c) $P = \frac{28}{46} = \frac{14}{23}$ (d) $P = \frac{18}{48} = \frac{3}{8}$

EXERCISE 1.5



Chance and compound events

1. Describe these events as impossible, very unlikely, unlikely, even chance, likely, very likely or certain.
(a) The sun will set in the west.
(b) A standard die is rolled and a 6 results.
(c) It will rain in Broome next summer.
(d) An ace is the first card dealt from a deck of cards.

2. Determine the approximate probability of these events using the categories 0, 0 to 0.5, 0.5, 0.5 to 1 and 1.

(a) Next Christmas day will fall on 25 December.

(b) A person selected at random will be left-handed.

(c) A double-headed coin will come down tails when tossed.

(d) The temperature in Hobart will be > 40°C on a summer day.

3. The numbers 1 to 15 are written on identical cards and placed face down. A card is turned over at random. Find the probability that the number is:

(a) An odd number.

(b) A number divisible by 3.

(c) A 2-digit number.

(d) A prime number.

4. The word *telephone* is spelled out by writing each letter on a separate card. The cards are then placed face down and mixed up. A card is selected at random. Find the probability that the letter is:

(a) The letter p.

(b) A vowel.

(c) One which appears more than once in the word.

5. Jim takes the four kings from the deck and places them face down on the table. He offers you a choice of any card. Find the probability that the card chosen is:

(a) The king of diamonds.

(b) A black king.

(c) The king of hearts or king of spades.

(d) A picture card.

6. Tim rolls a 12-sided die with faces numbered 1 to 12. What is the probability of obtaining:

(a) A 5?

(b) A number other than 5?

(c) A number not divisible by 4?

7. From a well-shuffled pack of 52 playing cards, one card is chosen at random. What is the probability that the card chosen is:

(a) The queen of hearts?

(b) Not the queen of hearts?

(c) Neither 4 nor 5?

(d) Not a black king?

(e) Not a heart?

(f) Not a picture card?

8. A target shooter knows from past performance that he has hit the bullseye on 45% of shots. If the shooter fires 60 shots, how many bullseyes are expected?

9. A raffle ticket is drawn from a box containing 500 tickets. Find the probability that the winning ticket is numbered at least 100.

10. A die is tossed. What is the probability that the number is:

(a) At least 3?

(b) At most 5?

11. Two dice are thrown and the sum of the two dice is recorded. Make up a 6 by 6 grid and write down the sum of the numbers from all possible combinations. How many possible outcomes are in the sample space?

Find the probability of:

(a) A sum of 2.

(b) A sum of 7.

(c) A sum > 5.

(d) A sum of at least 6.

(e) A sum of at most 6.

(f) A sum of 6 or 7.

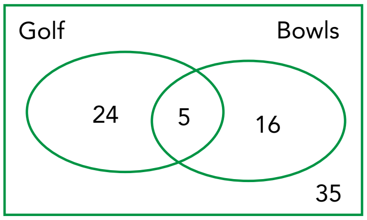
(g) A sum of 6 or 8.

(h) An even sum.

(i) A unique sum.

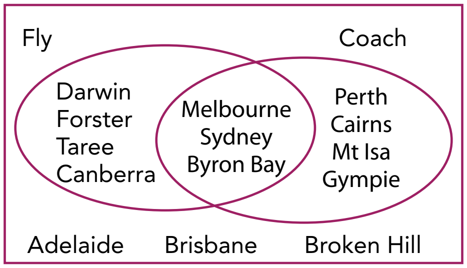
	1	2	3	4	5	6
1						
2						
3						
4						
5						
6						

12. Calculate the probability of the event described by referring to the Venn diagram. In a retirement home there are retirees that play golf and bowls. There are also people that do no sports or other sports like fishing. Shown here are the figures for golf, bowls and others (35). If someone is selected at random, find the probability he/she:



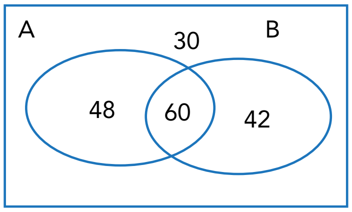
(a) Plays golf.
(b) Plays bowls.
(c) Plays both.
(d) Plays either golf or bowls (or both).
(e) Plays neither golf nor bowls.

13. A tour operator conducted a survey of favoured travel destinations and preferred mode of travel. If we selected one city/town from this list, what is the probability that:



(a) People would fly there?
(b) People would go there by coach?
(c) People would go there by either mode of travel?
(d) People would not fly there?
(e) They would only go there by coach?
(f) They would not go there by either mode?

14. Convert this Venn diagram into a two-way table. Label your rows and columns A, Not A, B and Not B.



(a) How many items are in the sample space?
(b) If one item is selected at random find the probability that it is not an A.
(c) If an A is selected at random, what is the probability it is not a B?
(d) If a B is selected at random, what is the probability it is also an A?
(e) If an item that is not a B is selected, what is the probability it is also not an A?
(f) For one selected item find P(A XOR B). (XOR = exclusive or meaning either one or the other but not both.)

15. Students must study either history or geography. Complete this table and answer the questions.

	Geography	**History**	**Totals**
Male	12		
Female			38
Totals	30	50	80

(a) What is the probability that a geography student is male?
(b) What is the probability that a boy studies history?
(c) What is the probability that a student picked at random studies geography?
(d) What is the probability that a student picked at random is female?
(e) What is the probability that a history student picked at random is female?
(f) What is the probability that a girl picked at random studies geography?
(g) What is the probability that a student picked at random is either a male geography student or a female history student?

1.6 Products and factors

Substitution

Since a variable stands for a number, we can substitute any number for the variables in an expression and calculate its value.

Example 1: Find the value of these expressions when $x = 5$; $y = -2$.

(a) $x + 6$ (b) $y - 5$ (c) $2y + 5$ (d) $4y^2$

Solution:

(a) $x + 6$
$= 5 + 6$
$= 11$

(b) $y - 5$
$= -2 - 5$
$= -7$

(c) $2y + 5$
$= 2 \times (-2) + 5$
$= -4 + 5$
$= 1$

(d) $4y^2$
$= 4 \times (-2)^2$
$= 4 \times 4$
$= 16$

Collecting like terms

Like terms are terms that contain the same letters exactly.

Example 1: (a) Add $7ab$ and $11ab$. (b) Add $3a$, $6b$ and $2a$.

Solution: (a) $7ab + 11ab = 18ab$ (b) $3a + 6b + 2a = 5a + 6b$

Example 2: Simplify $8x + 4y + 3x + 5y$.

Solution: $8x + 4y + 3x + 5y = (8x + 3x) + (4y + 5y) = 11x + 9y$

Multiplying variables

Example: Multiply the single variables together.

(a) $4 \times 5x$ (b) $ab \times pq$

(c) $3a \times (-4y) \times 7c$ (d) $-3d^2 \times (-5a^3)$

Solution:

(a) $4 \times 5x = 20x$

(b) $ab \times pq = abpq$

(c) $3a \times (-4y) \times 7c = -84acy$

(d) $-3d^2 \times (-5a^3)$
$= 15a^3d^2$

Applying the distributive law to the expansion of algebraic expressions

Example: Expand the brackets:

(a) $4(x - 8)$ (b) $3y(x - 5)$ (c) $-3y(5y - x)$

Solution:

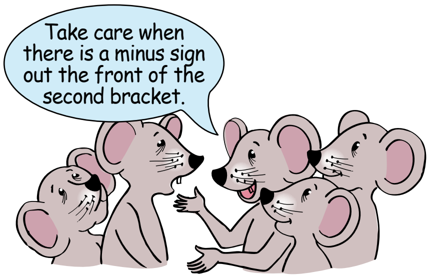
(a) $4(x - 8) = 4 \times x - 4 \times 8 = 4x - 32$

(b) $3y(x - 5) = 3y \times x - 3y \times 5 = 3xy - 15y$

(c) $-3y(5y - x) = -3y \times 5y - (-3y) \times x = -15y^2 + 3xy$

Note: The product of two negatives is +.

Expanding brackets and collecting like terms



Example: Expand $4(x - 2) - 2(x + 3)$.

Solution: $4(x - 2) - 2(x + 3) = 4x - 8 - 2x - 6 = 2x - 14$

Factorising algebraic expressions

Example: Factorise: (a) $3x + 15$ (b) $6a + 14$

Solution: (a) $3x + 15 = 3 \times x + 3 \times 5 = 3(x + 5)$

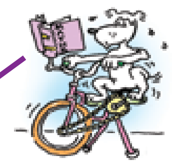
Common factor is 3.

(b) $6a + 14 = 3 \times 2 \times a + 2 \times 7 = 2(3a + 7)$

Common factor is 2.

Factorisation is the magic of algebra! It turns addition and subtraction into multiplication!

EXERCISE 1.6



Products and factors

1. Calculate the value of the following expressions if $t = 2$, $x = -3$, $y = 4$.

(a) $x + 16$ (b) $t - 4$ (c) $t + x$ (d) $3x - 2$
(e) $2t + 3y$ (f) $2t - 3x$ (g) $2y^2 + t$

2. Calculate the value of the following expressions if $p = 3$, $q = 4$, $r = -5$.

(a) $qr - 5$ (b) $pq + r$ (c) $q^2 + pr$ (d) $r^2 - 2pq$
(e) $p^2 - rq$ (f) $p^2 + q^2$ (g) $q^2 - r^2$

3. Add and subtract the like terms to give a single algebraic term.

(a) $4x + 7x$ (b) $9ab + 5ab$ (c) $3x^2 + 5x^2$
(d) $15a^3 + 6a^3$ (e) $7ax^2 + 2ax^2$ (f) $8b^2t + 3tb^2$
(g) $9x - 2x$ (h) $10bc - 8bc$ (i) $gx - 6gx$
(j) $9z^3 - 5z^3$ (k) $5tb^3 - 2b^3t$ (l) $-f^3 - 4f^3$

4. Simplify these algebraic expressions.

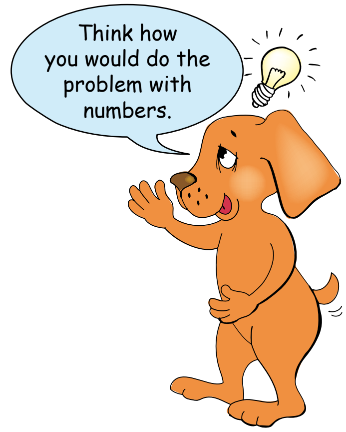
(a) $6a - 3a + 4a$ (b) $8g - 4g - g$ (c) $6by + 3by - 7by$ (d) $8z^3 - z^3 + 8z^3 - z^3$

5. Simplify these algebraic expressions by adding or subtracting like terms.

(a) $6y + 4a - 4y$ (b) $7g - 7h - 8g$ (c) $3by + 5cx - 2by$
(d) $3x^2 + 9y^2 + 7x^2$ (e) $7y + 6a - 4y + 3a$ (f) $9h - 9y - 11h - 7y$
(g) $9ax + 8xy - 2ax + 3xy$ (h) $5x^3 + 4t^3 - x^3 - 5t^3$

6. Simplify these expressions by multiplying the algebraic terms.

(a) $6 \times 2y$ (b) $10 \times (-4t)$ (c) $6y \times b$ (d) $-8q \times (-10)$
(e) $5x \times 2a$ (f) $3a \times 5b \times 4c$ (g) $-5d \times (-7a)$ (h) $3b \times (-y) \times a$
(i) $4x^2 \times 3a$ (j) $2y^3 \times 7b^2$ (k) $2a^2 \times 5b^3 \times 4c^3$ (l) $-6r \times (-2t) \times (-5f)$



7. Write an algebraic expression for the answer to these problems.

(a) How many km does a car travel in $2x^2$ hours at $7y^3$ km/h?

(b) What is the cost of $25a^2x^3$ kg of fertiliser at \$$2by^3$ per kg?

(c) How much fuel is pumped by a pump working at $6t$ litres per minute if it pumps for $5a^2$ minutes?

8. Perform these divisions.

(a) $\frac{8y}{4}$ (b) $\frac{-24b}{-8}$ (c) $\frac{40pq}{5}$

(d) $-18x \div (-6)$ (e) $40st \div (-10)$

9. Simplify these expressions by expanding the brackets.

(a) $6(a + 4)$ (b) $5(x - 6)$ (c) $7(3b - 2a)$ (d) $7y(y + 2)$

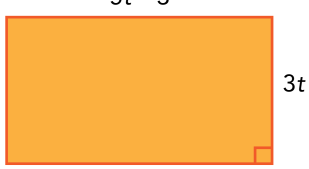
(e) $5y(3y - 2)$ (f) $4k(9 - 2k)$ (g) $-3y(y + 2)$ (h) $-7m(m - 6)$

(i) $-4q(q - 2)$ (j) $-5d(2 - d)$ (k) $3y(2y + 3)$ (l) $-7n(3n + 2)$

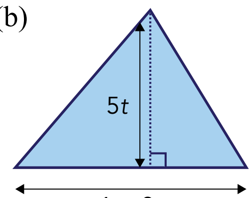
(m) $-3a(5 - 3a)$ (n) $-2x(8 - 3x)$ (o) $4y(2 - 5y)$

10. Calculate the area of these figures. Measurements are in centimetres.

(a)



(b)



11. Expand the brackets and collect like terms.

(a) $5(y + 4) + 2(y + 1)$ (b) $3(g - 2) + 2(g + 3)$ (c) $9(b - 2) + 3(b - 1)$

(d) $4(a + 3) - 2(a + 1)$ (e) $4(g - 5) - 3(g - 2)$ (f) $2(t - 5) - 2(t + 3)$

(g) $a(a + 3) + 4(a + 2)$ (h) $g(g - 5) + 3(g - 2)$ (i) $t(t - 5) + 2(t + 3)$

12. Expand the brackets and collect like terms.

(a) $y(y + 4) - 2(y + 1)$ (b) $x(x - 6) - 3(x - 2)$ (c) $g(g + 3) - 5(g - 4)$

(d) $y(3y + 5) + 3(2y + 4)$ (e) $g(2g - 5) + 3(g - 2)$ (f) $2t(t - 5) - 2(t + 3)$

(g) $5a(2a + 1) + 3a(4a + 3)$ (h) $3g(2g - 5) + 2g(g - 2)$ (i) $8b(2b - 5) - 5b(3b - 1)$

13. Factorise these expressions by finding the highest common factor.

(a) $5x + 15$ (b) $16a^2 + 8$ (c) $18t - 6$ (d) $6k - 21$ (e) $28 - 35g$

14. Verify these factors by checking that a substituted variable produces the same value. The first one is done for you.

Substitution	Expression	Factors	Value	Expression	Value
$x = 2$	$30x - 24$	$6(5x - 4)$	$6 \times (5 \times 2 - 4) = 36$	$30x - 24$	$30 \times 2 - 24 = 36$
$y = 3$	$40y + 8$			$40y + 8$	
$z = 4$	$16z - 8$			$16z - 8$	
$t = 2$	$15t^2 - 12$			$15t^2 - 12$	

15. Factorise these expressions by taking out the highest common factor.

(a) $k^3t + k^2t^2$ (b) $pqr - pqt$ (c) $mnp^3 - m^2np$

(d) $15en^3 + 12e^2n^2$ (e) $12abc - 18abz$ (f) $6gk^3t - 9g^2kt$

Here is an example.

$6a^3x + 2a^2x^5$

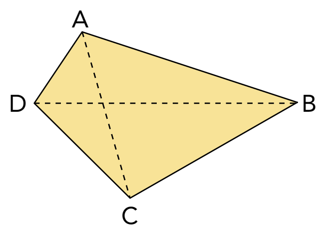
$= 2a^2x(3a + x^4)$

1.7 Special quadrilaterals

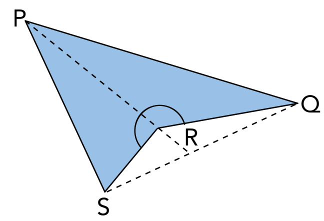
Recognising and naming quadrilaterals

A quadrilateral is a plane shape with four straight sides. There are many special quadrilaterals with additional properties, but all quadrilaterals are described by the given definition.

Convex quadrilaterals



Non-convex quadrilaterals



Properties of convex quadrilaterals

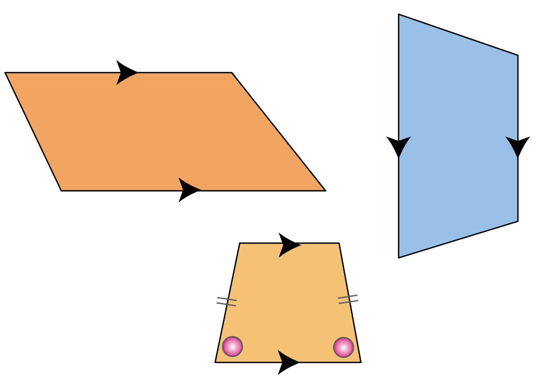
- Four unequal sides.
- Each internal angle < 180º.
- No sides necessarily equal.
- No angles necessarily equal.
- No necessary symmetry.
- Diagonals meet inside the quadrilateral.

Properties of non-convex quadrilaterals

- One angle > 180º.
- No sides necessarily equal.
- No angles necessarily equal.
- No symmetry necessary.
- One of the diagonals must be produced (extended) in order for them to meet (outside the quadrilateral).

The angle sum of any quadrilateral is 360°.

Trapezium: (Trapezoid in USA) A trapezium is a quadrilateral with 1 pair of opposite sides parallel. There is a special class of trapezium called an isosceles trapezium, which has the non-parallel pair of sides equal.

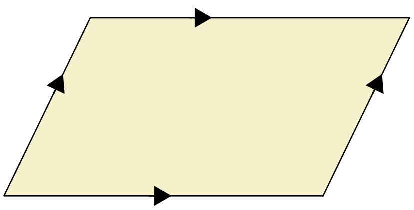


Isosceles trapezium

Properties of trapeziums

- All trapeziums are convex quadrilaterals.
- A trapezium has 2 opposite sides parallel.
- An isosceles trapezium has 2 non-parallel equal sides.
- Whereas in general a trapezium has no axis of symmetry, an isosceles trapezium has 1 axis of symmetry.

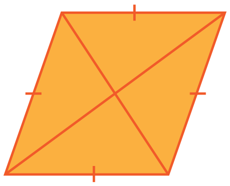
Parallelogram: A parallelogram is a quadrilateral with both pairs of opposite sides parallel.



Properties of parallelograms

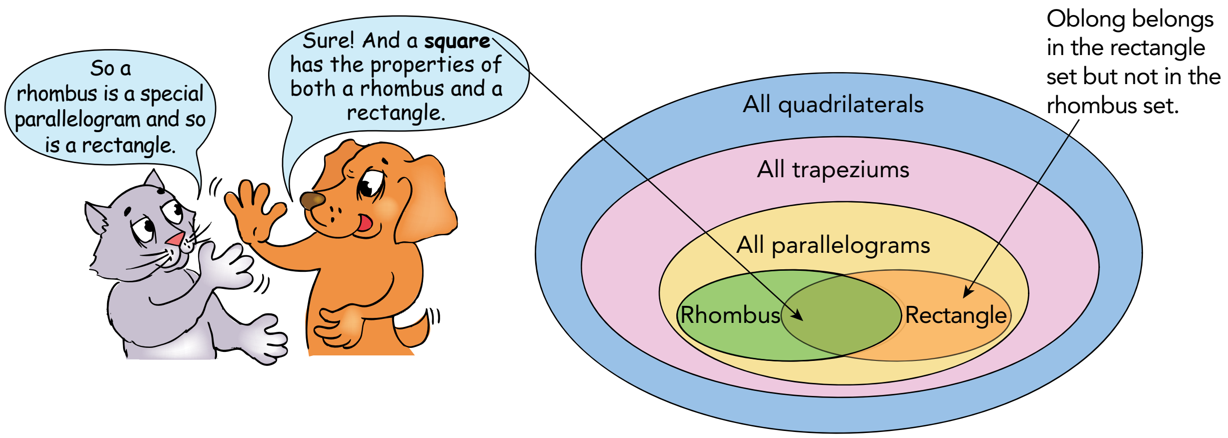
- Both pairs of opposite sides are parallel.
- Both pairs of opposite sides are equal.
- Both pairs of opposite angles are equal.
- The diagonals bisect each other.
- A parallelogram has rotational symmetry of order 2.

Rhombus: A parallelogram with adjacent sides equal.



Rectangle: A parallelogram with at least 1 right angle.

Rhombuses and rectangles are all parallelograms



Properties of rectangles and rhombuses

Rectangle:

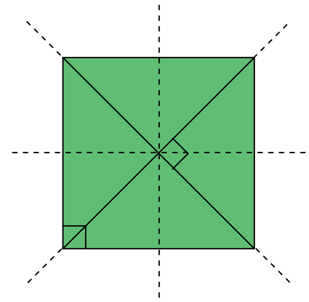
- Diagonals are equal.
- All angles are right angles.
- Two axes of symmetry parallel to the sides.
- Rotational symmetry of order 2.

Rhombus:

- Diagonals are perpendicular.
- All sides are equal.
- Two axes of symmetry along the diagonals.
- Rotational symmetry of order 2.
- Diagonals bisect the angles.

Square: A square is a quadrilateral with all of the properties of a rhombus and a rectangle together.

Like a rhombus, a square has:



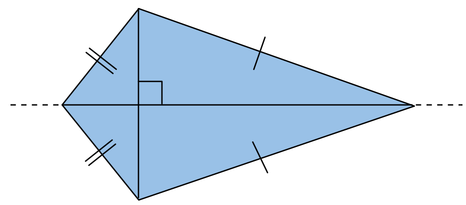
- All 4 sides equal.
- Diagonals are perpendicular.
- The diagonals are axes of symmetry.

Like a rectangle, a square has:

- All angles are right angles.
- Diagonals are equal.
- Two axes of symmetry parallel to the sides.

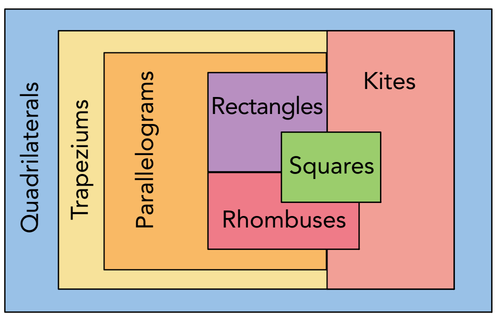
Kite: A kite is a quadrilateral with 2 pairs of adjacent sides equal. A kite has some properties of a rhombus.

Like a rhombus, a kite has:



- Diagonals perpendicular.
- Two pairs of adjacent side equal.
- One diagonal axis of symmetry.

However, unlike a rhombus, a kite is not a parallelogram.

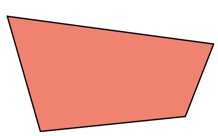
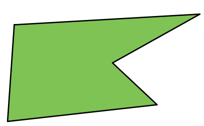
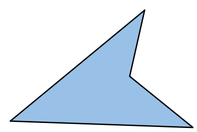
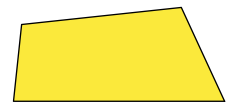


The Carroll diagram shows that all rectangles are parallelograms, while all kites are not. Rhombuses may be regarded as parallelograms or kites, while squares are specialised rectangles, rhombuses or kites.

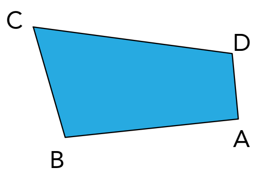
EXERCISE 1.7

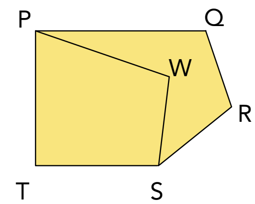
Special quadrilaterals

1. Identify the figures that are quadrilaterals and categorise them as *convex* or *non-convex* quadrilaterals.

(a)  (b)  (c)  (d) 

2. Write down the name of each quadrilateral in the diagram (there may be more than one).
Hint: How many quadrilaterals in the diagram?

(a) 

(b) 

Answers

Chapter 1 Year 8 Review

Getting started

1 D **2** C **3** B **4** B **5** C **6** C **7** B **8** A **9** B **10** D **11** C **12** A

1.1 Using the four operations with integers and rational numbers and using index notation with numbers

1 (a) –6 (b) –2 (c) –35 (d) –3 (e) 14 (f) 4 (g) 6 (h) –144 **2** (a) 16 (b) 64 (c) 121 (d) 0.09 (e) –27 (f) –64 (g) 1 (h) 1250 **3** (a) –3.6 (b) 3.3 (c) –0.44 (d) –0.93 (e) –0.4 (f) 0.4 (g) 0.16 (h) –0.008 **4** (a) $-\frac{1}{12}$ (b) $\frac{3}{10}$ (c) $\frac{16}{25}$ (d) $\frac{27}{64}$ (e) $-\frac{1}{6}$ (f) $-\frac{21}{4}$ or $-5\frac{1}{4}$ (g) $\frac{27}{16}$ or $1\frac{11}{16}$ (h) $-\frac{16}{15}$ or $-1\frac{1}{15}$ **5** (a) 6 (b) 18 (c) $x = \pm 16$ (d) $y = \pm 9$ **6** –22 **7** –5 and –6 **8** (a) 8 (b) 81 (c) –32 (d) –125 (e) 64 (f) –10 000 000 **9** (a) 2^4 (b) 6^2 (c) $(-2)^3$ (d) $(-10)^3$ **10** (a) 50 (b) 44 (c) –31 (d) 36 (e) 13 **11** (a) 5^{10} (b) 3^{50} (c) 8^2 (d) 2^4 (e) 17^2 **12** (a) 2^{15} (b) 3^8 (c) 10^{12} (d) 5^{198} (e) 3^9 **13** (a) 2^8 (b) 16^2 (c) 2^4 (d) 4^{50} (e) 10^4 **14** (a) 1 (b) 2^{198} (c) 7^6 (d) 10^{12} **15** (a) 2048 (b) 1024 (c) 8 (d) 4096 (e) 16

1.2 Working with real numbers – rational and irrational

1 (a) 0.5 (b) 0.25 (c) 0.2 (d) 0.375 (e) $0.\dot{3}$ (f) $0.\dot{1}4285\dot{7}$ (g) $0.\dot{2}$ (h) $0.8\dot{3}$ **2** (a) $\frac{3}{5}$ (b) $\frac{6}{25}$ (c) $\frac{5}{8}$ (d) $\frac{7}{80}$ (e) $\frac{1}{400}$ **3** (a) $\frac{8}{9}$ (b) $\frac{53}{99}$ (c) $\frac{6}{11}$ (d) $\frac{541}{999}$ (e) $\frac{334}{3333}$ **4** (a) $\frac{8}{15}$ (b) $\frac{5}{12}$ **5** (a) $0.\dot{0}\dot{9}$ (b) $0.\dot{1}\dot{8}$ (c) $0.\dot{2}\dot{7}$ (d) $0.\dot{3}\dot{6}$. Two-digit repeating decimals with the digits formed by multiplying numerator by 9. **6** (a) $0.\dot{0}7692\dot{3}$ (b) $0.\dot{1}5384\dot{6}$ (c) $0.\dot{2}3076\dot{9}$ (d) $0.\dot{3}0769\dot{2}$ **7** (a) See Questions 5 and 6. (b) Yes (c) nine **8** (a) 1 (b) 1 (c) 1.5 (d) –0.25 (e) 1.375 (f) 1.429688, 1.407685 (1.41) Yes. **9** (a) Irrational (b) Rational (c) Rational (d) Irrational **10** (a) Irrational (b) Irrational (c) Rational (d) Irrational (e) Rational (f) Irrational (g) Irrational (h) Rational (i) Irrational (j) Rational **11** Diameter **12** D

1.3 Calculating percentage change, ratio and rates

1 (a) \$37.50 (b) \$315 mL (c) 210 cm (d) 4.32 L **2** (a) 200 kg (b) \$6.75 (c) 1275 g (d) 21 min **3** (a) 0.42 kg (b) 1.35 m (c) 3.6 km (d) 0.27 t **4** \$25.50 **5** $37\frac{1}{2}$% of \$600 by \$10 **6** (a) 40% (b) 20% (c) 37.5% (d) 7.5% (e) 25% (f) 85% (g) 20% (h) 36% (i) 18.75% (j) 20% **7** 64% boys, 36% girls **8** 50% copper, 25% zinc, 25% nickel **9** 12.5% white, 25% yellow, 62.5% green **10** (a) \$54 (b) 11.2 kg (c) 2.76 m (d) \$420 (e) 24 kg (f) 2.56 L **11** 8160 **12** 1000 L **13** \$800 **14** \$25.60. Yes. **15** 25% **16** 2.8 m^3 cement, 5.6 m^3 sand, 8.4 m^3 gravel **17** 20 min A, 40 min B **18** 660 mm, 880 mm, 1100 mm **19** (a) 11.46 cm, 18.54 cm (b) 26.18 cm **20** (a) $53\frac{1}{3}$ km/h (b) 2 km/h (c) 8 cm/min (d) 0.75 mm/h **21** (a) 1.3 runs/ball (b) 130 **22** 300 years **23** (a) A\$1.053 (b) A\$26.32 **24** (a) 13 km/h (b) 3 km/h (c) 4.875 km/h

1.4 Congruence

1 a, b, d **2** (a) SSS (b) RHS (c) AAS (d) SAS (e) SAS (f) SSS **3** (a) No, angle not included. (b) No, not corresponding side. (c) Yes, SAS **4** (a) ΔABC, ΔHIG (RHS) (b) ΔABC, ΔIGH (SSS) (c) ΔABC, ΔIGH (AAS) (d) ΔABC, ΔGIH (SAS) (e) ΔABC, ΔDFE, ΔIHG (AAS) (f) ΔABC, ΔFED, ΔGHI (AAS) **5** AP = BP (given), CP = DP (given), ∠APC = ∠BDP (vertically opposite angles equal), ∴ ΔACP ≡ ΔBDP (SAS). ∴ AC = BD

1.5 Chance and compound events

1 (a) Certain (b) Unlikely (c) Very likely (d) Very unlikely **2** (a) 1 (b) 0 to 0.5 (c) 0 (d) 0 to 0.5 **3** (a) $\frac{8}{15}$ (b) $\frac{1}{3}$ (c) $\frac{2}{5}$ (d) $\frac{2}{5}$ **4** (a) $\frac{1}{9}$ (b) $\frac{4}{9}$ (c) $\frac{1}{3}$ **5** (a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) $\frac{1}{2}$ (d) 1 **6** (a) $\frac{1}{12}$ (b) $\frac{11}{12}$ (c) $\frac{3}{4}$ **7** (a) $\frac{1}{52}$ (b) $\frac{51}{52}$ (c) $\frac{11}{13}$ (d) $\frac{25}{26}$ (e) $\frac{3}{4}$ (f) $\frac{10}{13}$ **8** 27 **9** $\frac{401}{500}$ **10** (a) $\frac{2}{3}$ (b) $\frac{5}{6}$ **11** 36 (a) $\frac{1}{36}$ (b) $\frac{1}{6}$ (c) $\frac{13}{18}$ (d) $\frac{13}{18}$ (e) $\frac{5}{12}$ (f) $\frac{11}{36}$ (g) $\frac{5}{18}$ (h) $\frac{1}{2}$ (i) $\frac{1}{18}$ **12** (a) $\frac{29}{80}$ (b) $\frac{21}{80}$ (c) $\frac{1}{16}$ (d) $\frac{9}{16}$ (e) $\frac{7}{16}$ **13** (a) $\frac{1}{2}$ (b) $\frac{1}{2}$ (c) $\frac{3}{14}$ (d) $\frac{1}{2}$ (e) $\frac{2}{7}$ (f) $\frac{3}{14}$ **14** (a) 180 (b) $\frac{2}{5}$ (c) $\frac{4}{9}$ (d) $\frac{10}{17}$ (e) $\frac{5}{13}$ (f) $\frac{1}{2}$ **15** (a) $\frac{2}{5}$ (b) $\frac{5}{7}$ (c) $\frac{3}{8}$ (d) $\frac{19}{40}$ (e) $\frac{2}{5}$ (f) $\frac{9}{19}$ (g) $\frac{2}{5}$

	Geography	History	Totals
Male	12	30	42
Female	18	20	38
Totals	30	50	80