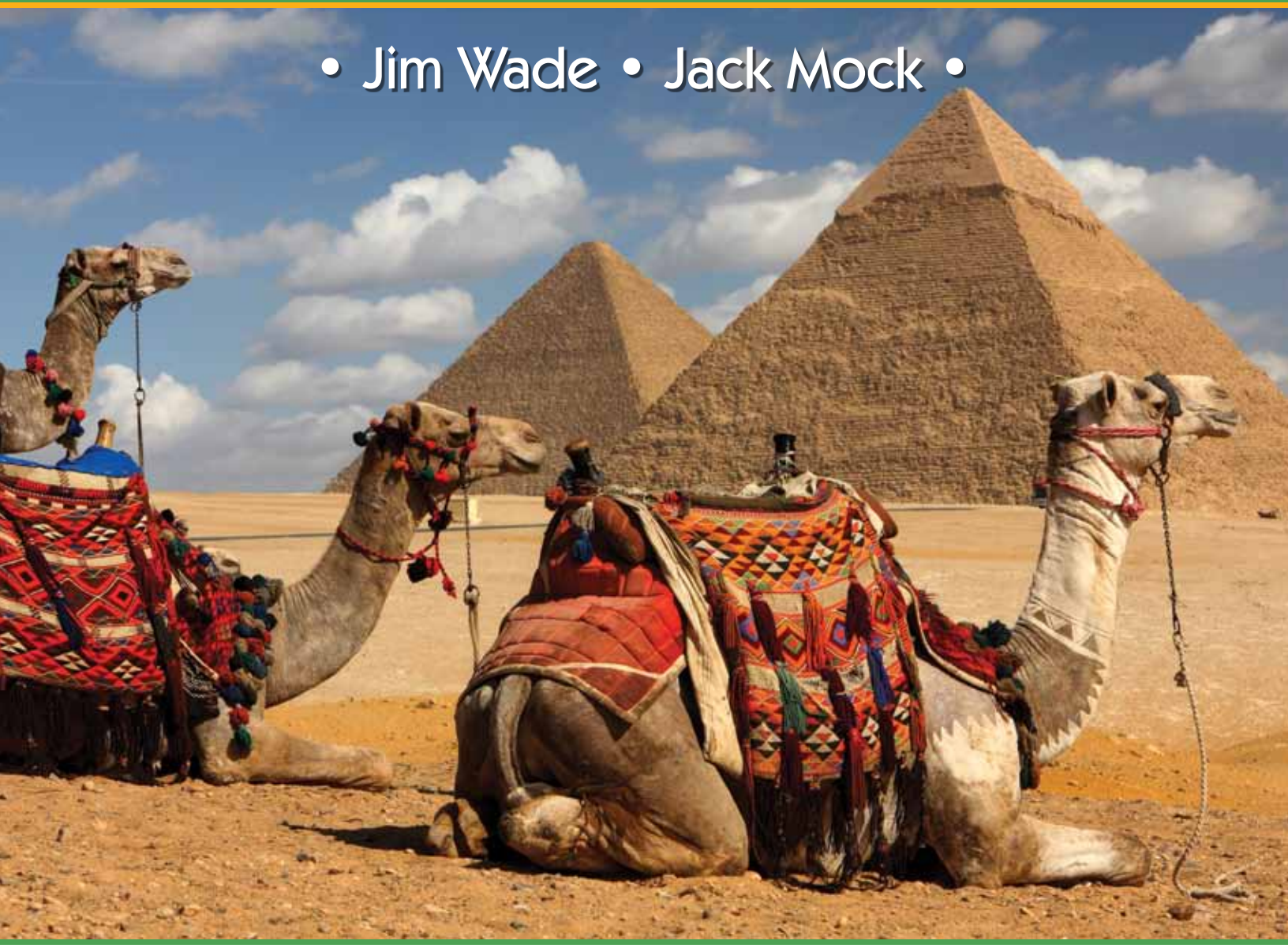
NATIONAL MATHS

YEAR 7

• Jim Wade • Jack Mock •



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Contents

Chapter 1
The Laws of Arithmetic

Syllabus

Apply the associative, commutative and distributive laws to aid mental and written computation. (ACMNA151)

Compare, order, add and subtract integers. (ACMNA280)

KEY SKILLS AND KNOWLEDGE

By the end of this chapter you should be able to:

- Use the associative and commutative laws for addition. (1.1)
- Use the associative and commutative laws for multiplication to aid calculation. (1.1)
- Use an appropriate non-calculator method to divide 2-digit and 3-digit numbers by a 2-digit number. (1.2)
- Check estimates of answers obtained by written methods with a calculator. (1.2)
- Solve number puzzles such as magic squares and arithmagons. (1.3)
- Compare different numbers for size and plot them on a number line. (1.4)
- Plot both positive and negative numbers on a number line. (1.5)
- Add and subtract both positive and negative numbers. (1.6)
- Perform mathematical operations in the correct order and use the distributive law. (1.7)
- Use basic mathematical symbols. (1.8)

WHO AM I?

I am an integer between 10 and 99.
The sum of my digits is 7.
I am even.
My units digit is one greater than my tens digit. Who am I?

1.3 Solving number puzzles

Magic squares

A **magic square** is one in which all the rows, columns and diagonals have the same sum.

For example, the magic square shown is of the order 3 (3 columns, 3 rows) and has the 'magic' number 15.

The Lo-Shu (scroll from the River Lo) square first appeared on an ancient Chinese tablet in 2200 BCE, in the reign of Emperor Yu, and is the earliest record of a magic square. In legend, it is said to have appeared in dots on a tortoise's shell following a flood.

4	9	2
3	5	7
8	1	6

EXERCISE 1.3

Solving number puzzles

1. Complete these magic squares.

(a)

4		8
	7	
		10

(b)

2	7	6
	5	
4		

(c)

20	15	22
21		

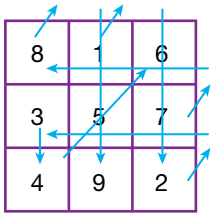
(d)

	3	8
	5	
2		

2. Use each of the numbers 1, 2, 3, ... 16 once only to complete this 4 × 4 magic square.

Hint: What is the sum of all the integers from 1 to 16? What is the sum of one row or column?

		7	
15			
9	5	16	
8		1	13



Here is a method for completing odd-sided magic squares.

1. Start at the top in the middle with 1.
2. Move diagonally up to the right for the next number.
3. If you move outside the square, slide to the far end of the adjacent column or row.
4. If rules 2 and 3 don't apply then go below the current square.

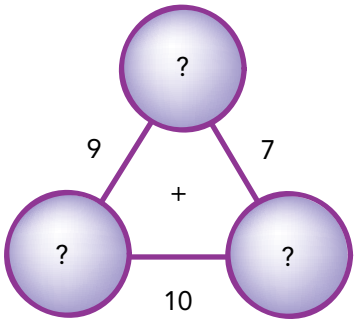
3. Have a go at completing this 5 × 5 magic square with the numbers from 1 to 25 using the above method.

Try a 7 × 7 square now you know how easy it is.

		1		

Arithmagons

Example: Find the missing numbers, given that all the numbers on the vertices of the triangle add up to the number given on the line joining them.



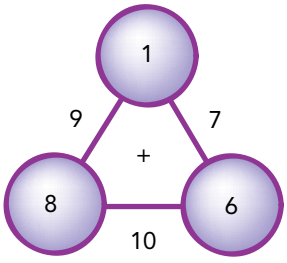
Solution: We could use the guess and check strategy.

Now 9 = 8 + 1 or 7 + 2 or 6 + 3 or 5 + 2 or 4 + 5

7 = 6 + 1 or 5 + 2 or 4 + 3

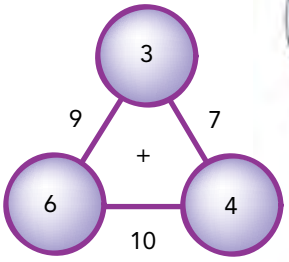
10 = 9 + 1 or 8 + 2 or 7 + 3 or 6 + 4

Let us try 9 = 8 + 1



8 + 6 ≠ 10

Try 9 = 6 + 3



6 + 4 = 10

Chapter 4
Fractions

Syllabus

Compare fractions using equivalence. Locate and represent fractions and mixed numerals on a number line. (ACMNA152)

Solve problems involving addition and subtraction of fractions, including those with unrelated denominators. (ACMNA153)

Multiply and divide fractions using efficient written strategies and digital technologies. (ACMNA154)

Express one quantity as a fraction of another, with and without the use of digital technologies. (ACMNA155)

KEY SKILLS AND KNOWLEDGE

By the end of this chapter you should be able to:

- Find equivalent fractions. (4.1)
- Write fractions with the same denominator to enable addition and subtraction. (4.1)
- Reduce a fraction to its simplest equivalent form by cancelling. (4.1)
- Express improper fractions as mixed numbers and vice versa. (4.2)
- Locate fractions and mixed numbers on a number line. (4.2)
- Express one quantity as a fraction of another. (4.2)
- Add and subtract fractions. (4.3)
- Add and subtract mixed numbers using written and calculator methods. (4.4)
- Subtract a fraction from a whole number. (4.4)
- Solve problems involving addition and subtraction of fractions. (4.4)
- Multiply and divide fractions and mixed numbers. (4.5)
- Solve problems involving fractions. (4.6)

INVESTIGATION

Take a simple sum of fractions, say $\frac{1}{3}+\frac{1}{4}=\frac{4}{12}+\frac{3}{12}=\frac{7}{12}$.

Now investigate the possibility of finding two other fractions in their lowest terms (not twelfths) which sum to $\frac{7}{12}$.

GROUP ACTIVITY

Ask two people in a group to nominate one fraction each between zero and one.

Find any other fraction that lies between the two nominated fractions (see Exercise 4.3 Question 9). You can use a calculator with a fraction key to help you. Now take the new (middle) fraction and find another fraction which lies between this one and the previous lower fraction. Keep passing the answer around the group, always finding a fraction between the new one and the smaller of the original two fractions. Answer this question: 'How many fractions lie in between any given pair of fractions?'

4.5 Multiplying and dividing fractions and mixed numbers

Example 1: Finding a fraction of a quantity.

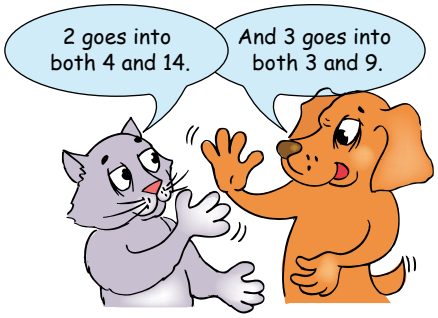
Find $\frac{2}{3}$ of 12. This means $\frac{2}{1\,\cancel{3}}\times\cancel{12}^{\,4}=8$

Example 2: Multiplying fractions (finding a fraction of a fraction).

$\frac{2}{3}\times\frac{4}{5}=\frac{2\times4}{3\times5}=\frac{8}{15}$

Example 3: Multiplying with cancelling.

$\frac{\cancel{4}^{\,2}}{\cancel{9}_{\,3}}\times\frac{\cancel{3}^{\,1}}{\cancel{14}_{\,7}}=\frac{2}{21}$



Using a calculator

We can use a fraction calculator to multiply $1\frac{1}{2} \times 2\frac{3}{4}$.

Using the fraction key enter 1 $a\frac{b}{c}$ 1 $a\frac{b}{c}$ 2 × 2 $a\frac{b}{c}$ 3 $a\frac{b}{c}$ 4 =

Answer: $4\frac{1}{8}$

Similarly to divide $3\frac{2}{3} \div 2\frac{1}{2}$:

Enter 3 $a\frac{b}{c}$ 2 $a\frac{b}{c}$ 3 ÷ 2 $a\frac{b}{c}$ 1 $a\frac{b}{c}$ 2 =

Answer: $1\frac{7}{15}$

EXERCISE 4.5

Multiplying and dividing fractions and mixed numbers

1. Calculate these quantities.

(a) $\frac{3}{4}$ of \$16 (b) $\frac{2}{3}$ of 24 hours (c) $\frac{4}{5}$ of 60 seconds (d) $\frac{3}{5}$ of 20 litres

(e) $\frac{2}{5}$ of 600K (f) $\frac{5}{6}$ of one dozen (g) $\frac{3}{8}$ of 12 hours (h) $\frac{3}{4}$ of \$18

(i) $\frac{4}{5}$ of 1000 metres (j) $\frac{1}{6}$ of 15 minutes

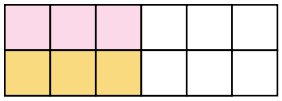
2. Multiply these fractions by selecting the required portion of the squares.

(a) (i) What fraction of the rectangle is coloured in?

(ii) What fraction of the coloured part is pink?

(iii) What fraction of the whole rectangle is pink?

(iv) Explain from the diagram the answer to $\frac{1}{2}$ of $\frac{1}{2}$.

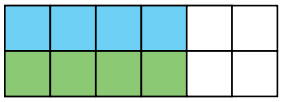


(b) (i) What fraction of the rectangle is coloured in?

(ii) What fraction of the coloured part is green?

(iii) What fraction of the whole rectangle is green?

(iv) Explain from the diagram the answer to $\frac{1}{2}$ of $\frac{2}{3}$.



(c) (i) What fraction of the rectangle is coloured in?

(ii) What fraction of the coloured part is purple?

(iii) What fraction of the whole rectangle is purple?

(iv) Explain from the diagram the answer to $\frac{1}{3}$ of $\frac{3}{4}$.

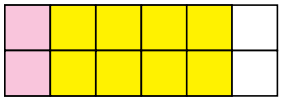
(v) Explain from the diagram the answer to $\frac{2}{3}$ of $\frac{3}{4}$.

(d) (i) What fraction of the rectangle is coloured in?

(ii) What fraction of the coloured part is yellow?

(iii) What fraction of the rectangle is yellow?

(iv) Explain from the diagram the answer to $\frac{4}{5}$ of $\frac{5}{6}$.



3. Verify your answers to Question 2 by calculating these products on a fraction calculator.

(a) $\frac{1}{2} \times \frac{1}{2}$ (b) $\frac{1}{2} \times \frac{2}{3}$ (c) $\frac{2}{3} \times \frac{3}{4}$ (d) $\frac{4}{5} \times \frac{5}{6}$

HOW MUCH DO YOU KNOW?

You are nearly at the end of the chapter. Check that you are able to do the following.

Find equivalent fractions (4.1)

- Find fractions equivalent to $\frac{3}{4}$ with denominators 8, 12, 20.

  Solution:
  $\frac{3}{4} = \frac{6}{8} = \frac{9}{12} = \frac{15}{20}$

Write fractions with the same denominator to enable addition and subtraction (4.1)

- Write the fractions $\frac{5}{8}$, $\frac{3}{5}$ with the same denominator.

  Solution:

  Find the LCM of the denominators 8 and 5 = 40

  Change fractions to denominator 40: $\frac{5}{8} \times \frac{5}{5} = \frac{25}{40}$, $\frac{3}{5} \times \frac{8}{8} = \frac{24}{40}$

Reduce a fraction to its simplest equivalent form by cancelling (4.1)

- Cancel $\frac{18}{30}$ to its simplest form.

  Solution:

  Find the HCF of 18 and 30 = 6. Cancel $\frac{18}{30} = \frac{3 \times \cancel{6}}{5 \times \cancel{6}} = \frac{3}{5}$

Express improper fractions as mixed numbers and vice versa (4.2)

- (a) Change $\frac{23}{4}$ to a mixed number.

  (b) Change $5\frac{3}{10}$ to an improper fraction.

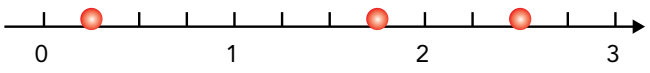
  Solutions:

  (a) 4 divides into 23 5 times with remainder 3.
  $\frac{23}{4} = 5\frac{3}{4}$

  (b) 5 is $\frac{50}{10}$.
  So $5\frac{3}{10} = \frac{50}{10} + \frac{3}{10} = \frac{53}{10}$

Locate fractions and mixed numbers on a number line (4.2)

Locate the numbers $\frac{1}{4}$, $1\frac{3}{4}$, $2\frac{1}{2}$ on a number line.



Express one quantity as a fraction of another (4.2)

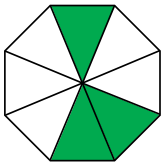
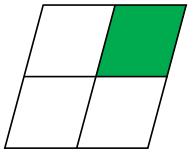
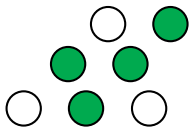
- Express 12 kg as a fraction of 28 kg and write the fraction in its simplest terms.

  Solution: $\frac{12}{28} = \frac{3 \times 4}{7 \times 4} = \frac{3}{7}$

  Note: Both quantities must be in the same units (kg).

CHAPTER 4 DIAGNOSTIC TEST

1. What fraction is the green part of the whole? **4.1**

(a)  (b)  (c) 

2. Cancel these fraction to their lowest terms. **4.1**

(a) $\frac{6}{8}$ (b) $\frac{6}{9}$ (c) $\frac{30}{36}$

3. What fraction in its lowest terms is the part of the whole described? **4.1**

(a) Luke birdied 12 of the 18 holes on the golf course.

(b) Australia won 16 gold medals from a total of 58 medals.

(c) Emma spent $24 of her $40 birthday gift on a CD.

4. Change these mixed numbers to improper fractions. **4.2**

(a) $3\frac{1}{3}$ (b) $2\frac{5}{8}$ (c) $5\frac{9}{10}$

5. Change these improper fractions to mixed numbers. **4.2**

(a) $\frac{3}{2}$ (b) $\frac{15}{4}$ (c) $\frac{40}{6}$

6. State true or false for each of these. **4.2**

(a) $2\frac{3}{4} < \frac{9}{4}$ (b) $\frac{4}{13} > \frac{4}{15}$ (c) $4\frac{5}{8} = \frac{44}{8}$

7. (a) $\frac{1}{4} + \frac{2}{3} =$ (b) $\frac{3}{4} - \frac{1}{3} =$ (c) $\frac{2}{3} + \frac{5}{8} =$ **4.3**

8. (a) $\frac{3}{4} + \frac{1}{8} =$ (b) $\frac{5}{6} - \frac{5}{9} =$ (c) $\frac{7}{12} - \frac{5}{18} =$ **4.3**

9. Arrange these fractions in ascending order (increasing from smallest to largest). **4.3**

(a) $\frac{1}{3}, \frac{1}{4}, \frac{1}{6}$ (b) $\frac{5}{8}, \frac{7}{12}, \frac{3}{4}$ (c) $\frac{9}{15}, \frac{7}{10}, \frac{2}{3}$

10. (a) $2\frac{1}{4} + 3\frac{1}{2} =$ (b) $3\frac{2}{5} + 1\frac{3}{8} =$ (c) $2\frac{5}{8} + 3\frac{5}{6} =$ **4.4**

11. (a) $5\frac{3}{4} - 2\frac{1}{2} =$ (b) $3\frac{2}{3} - 1\frac{1}{4} =$ (c) $4\frac{1}{5} - 2\frac{3}{4} =$ **4.4**

12. (a) $\frac{1}{2}$ of $\frac{1}{3} =$ (b) $\frac{2}{3} \times \frac{4}{5} =$ (c) $\frac{2}{3} \times \frac{9}{20} =$ **4.4**

13. (a) $1\frac{1}{2} \times \frac{1}{4} =$ (b) $2\frac{1}{2} \times 3\frac{1}{4} =$ (c) $4\frac{1}{5} \times 2\frac{1}{7} =$ **4.5**

14. (a) $\frac{3}{4} \div \frac{1}{4} =$ (b) $\frac{2}{3} \div \frac{5}{8} =$ (c) $\frac{4}{5} \div \frac{8}{15} =$ **4.5**

15. (a) $1\frac{1}{2} \div \frac{1}{4} =$ (b) $2\frac{2}{3} \div 1\frac{1}{4} =$ (c) $5\frac{1}{4} \div 2\frac{5}{8} =$ **4.5**

16. Calculate the required fraction of these quantities. **4.5**

(a) $\frac{3}{4}$ of $20 (b) $\frac{4}{5}$ of 3 minutes (c) $\frac{7}{8}$ of 1 km

17. Calculate the required fraction of these quantities. **4.5**

(a) $\frac{2}{3}$ of $60 (b) $\frac{3}{5}$ of 250 metres (c) $\frac{5}{8}$ of 2 dozen eggs

Chapter 7
Algebra

Syllabus

Introduce the concept of variables as a way of representing numbers using letters. (ACMNA175)

Create algebraic expressions and evaluate them by substituting a given value for each variable. (ACMNA176)

Extend and apply the laws and properties of arithmetic to algebraic terms and expressions. (ACMNA177)

KEY SKILLS AND KNOWLEDGE

By the end of this chapter you should be able to:

- Use letters to represent numbers. (7.1)
- Create and evaluate algebraic expressions. (7.2)
- Apply the laws of arithmetic to simplify algebraic expressions. (7.3)
- Use the four arithmetic operations with algebra. (7.4)
- Recognise the role of grouping symbols and the different meanings of expressions. (7.5)
- Connect algebra with the commutative and associative properties of arithmetic. (7.6)
- Interpret statements involving algebraic symbols in other contexts. (7.7)



GETTING STARTED

1. In algebra, if x stands for an unknown number, then one more than x is:
(A) y (B) $x + 1$
(C) $x - 1$ (D) Depends on what x is.

2. In algebra, one less than x is written:
(A) w (B) $x - 1$ (C) $0x$ (D) $1 - x$

3. If $x = 3$ then $x + 1$ equals:
(A) y (B) 4 (C) 6 (D) 31

4. If $y = 4$ then $y = 1$ equals:
(A) 3 (B) $3y$ (C) x (D) -4

5. When a letter stands for a number that could be many different values it is called a:
(A) Variable. (B) Changeable. (C) Alterable. (D) Valuable.

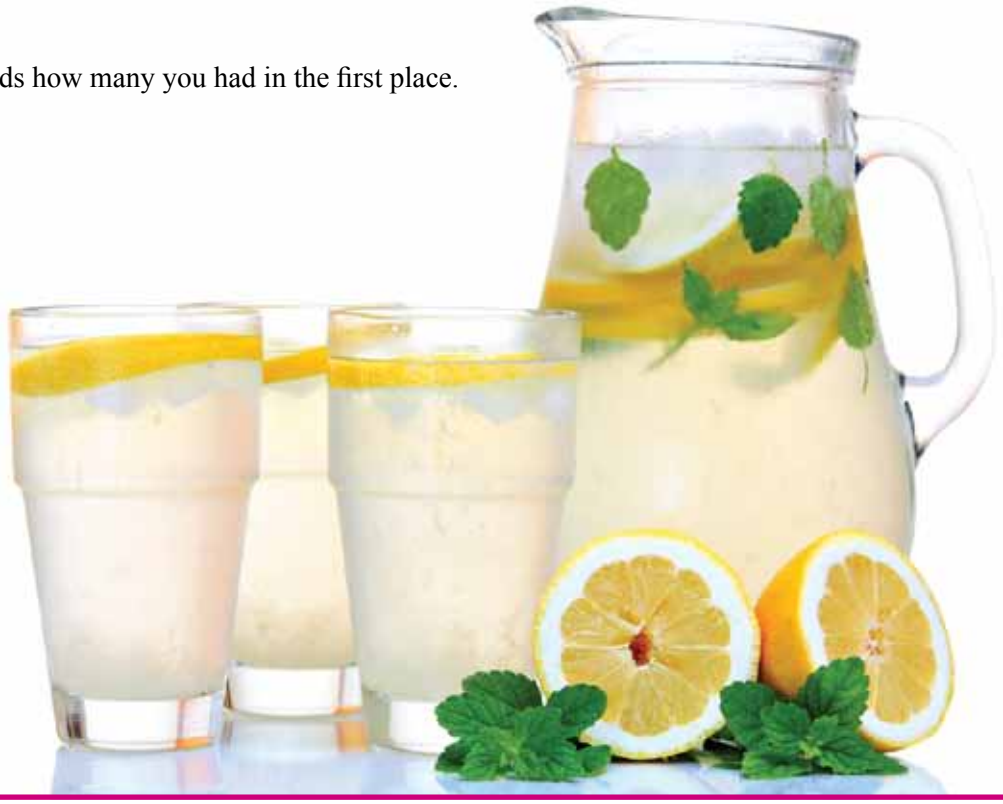
6. If $x = 7$ then $2x$ equals:
(A) 14 (B) 27 (C) 49 (D) $14x$

7. If $x = 12$ then $x + 2$ equals:
(A) 24 (B) 6 (C) $6x$ (D) None of these.

8. If $x = 8$ then y equals:
(A) 9 (B) $x + 1$ (C) $2x$ (D) Unknown.

9. If $x + 1 = 3$ then x must be:
(A) 2 (B) 4 (C) 13 (D) One before y.

10. If I had x bottles of lemonade in the fridge and I drank one, then the number of bottles left in the fridge would be:
(A) Still x.
(B) $x - 1$
(C) It depends how many you had in the first place.
(D) w



7.7 Using algebraic symbols in other contexts

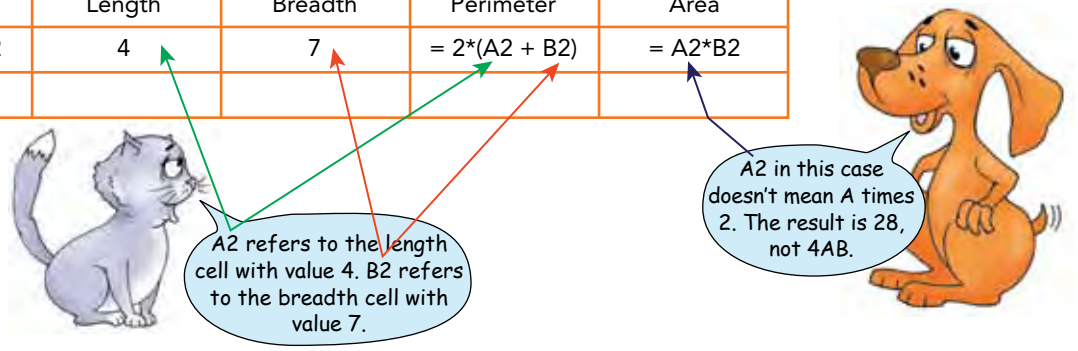
Sometimes algebra is used to represent numbers in a different way to what we have seen so far. For example, in a computer spreadsheet calculator there are different symbols for operations and formulas are constructed from their position in the grid.

Arithmetic symbol	Spreadsheet symbol	Example
+	+	3+4
–	-	4-3
×	*	4*3
÷	/	20/5
2^3	^	2^3

Spreadsheets

In a computer spreadsheet the value of a variable is found by referring to a cell in the spreadsheet. The cell has a name according to the column and row it belongs to.

	A	B	C	D
1	Length	Breadth	Perimeter	Area
2	4	7	= 2*(A2 + B2)	= A2*B2
3				



Computer programming

The instruction X3 = Y3 + 5 means:

Fetch the value of Y3. (23)

Add 5 to it. (28)

Store the result in X3.

Y3 = 23

X3 = 28

The instruction Y3 = Y3 + 5 means:

Fetch the value of Y3. (23)

Add 5 to it. (28)

Store this in Y3. (Note the original value of Y3 is lost.)

Y3 = 28

This statement would be meaningless in the context of ordinary algebra. It would say that there is some number X when multiplied by 3 (X3) and added to 5 (X3+5) gives the same number as when you multiplied it by 3 (X3), i.e. adding the 5 makes no difference.

Class property languages

Modern computer languages have variables that are subclasses of other variables.

For example, TABLEA may have columns X and Y while TABLEB also has columns X and Y. To refer to the different Xs and Ys they are separated from their table name by a dot. This allows the programmer to use the same variables X and Y in different contexts without getting them mixed up.

	Table A		Table B	
	X	Y	X	Y
1	7	11	22	55
2	8	12	23	56
3	9	13	24	57

So TABLEA.X(3) + TABLEB.X(3) = 9 + 24 = 33

EXERCISE 7.7

Using algebraic symbols in other contexts

1. Calculate the result in column D when these values and formulas are placed in a spreadsheet. Note that the normal order of operations applies, and brackets take precedence with powers being done before multiplication and division while addition and subtraction are last.

	A	B	C	D
1	First	Second	Third	Result
2	4	8	5	=A2*B2*C2
3	5	4	12	=A3*C3/B3
4	6	5	4	=B2^2-A4*C4
5	9	8	3	=A5/C5*B5
6	3	9	7	=(A6+C6)*B6
7	1	0	9	=(C7+B6-A5)*B7

2. Calculate the final values of x and y given the initial values of $x = 5$ and $y = 6$.

(a) $y = x\text{^}2 + 10$

(b) $y = x\text{^}2 + y\text{^}2$

(c) $x = x*y + x$

(d) $x = x*(x + y)$

(e) $y = (2*x + 3*y)/7$

3. Write a formula for the spreadsheet that will calculate the area of a triangle with given base and height. Place the formula in cell C2.

	A	B	C
1	Base	Height	Area of triangle
2	4	7	
3			

Hint 1: All spreadsheet formulas begin with '='.

Hint 2: The area of a rectangle is found by multiplying height by width.

Hint 3: Since a triangle may be considered to be half a rectangle, the area of a triangle is found by multiplying the height by the width (base) and halving the result.

4. Work out the values of these variables from the given table of class variables.

	Rectangle		Triangle	
	Height	Width	Height	Width
1	6.5	4	9.5	12
2	7.5	6	10.5	14
3	8.5	8	11.5	16

(a) Rectangle.Height(1)*Rectangle.Width(1)

(b) Rectangle.Height(3)*Rectangle.Width(3)

(c) Triangle.Height(2)*Triangle.Width(2)/2

(d) Triangle.Height(3)*Triangle.Width(3)/2

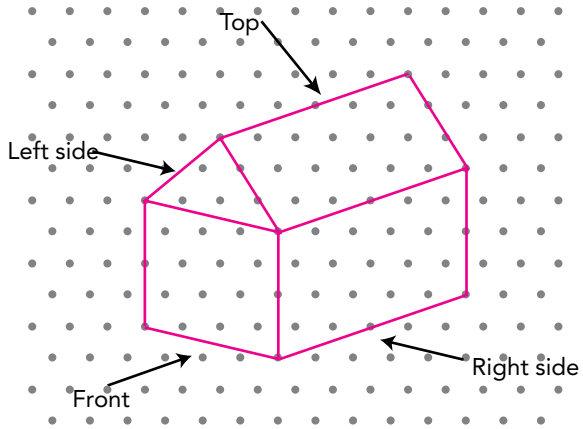
(e) (Rectangle.Height(1)+Rectangle.Width(1))*2

13.4 Drawing solids from different perspectives

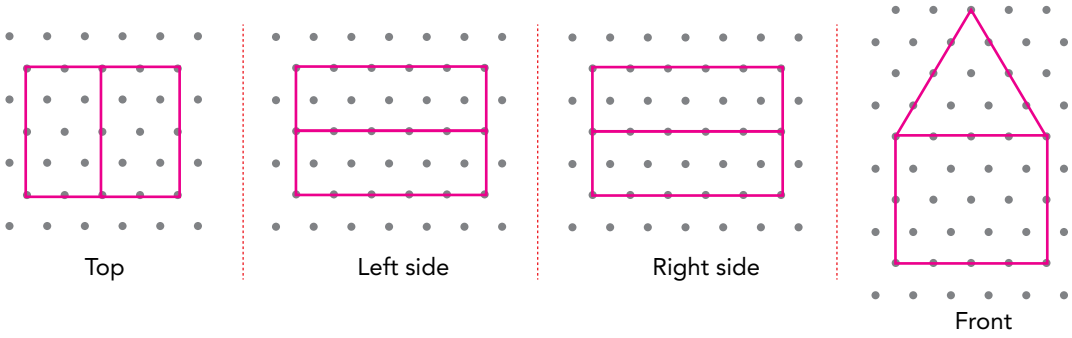
As we approach someone's house we see the **front view**. If we look at the house from the neighbour's view it would be the **side view** and if we approached the house from the rear it would be the **back view**.

An architect's floor plan of a house could be considered as the **top view** of the house (i.e. looking at the house from above). We can put all the views together to describe fully what the solid looks like.

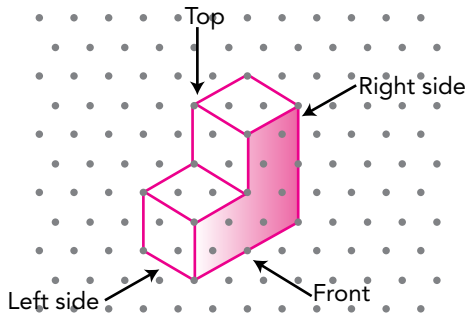
Example 1: Draw the top, left side, right side and front views of this house.



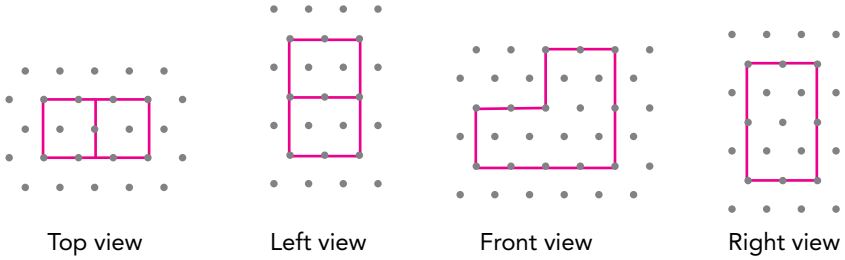
Solution:



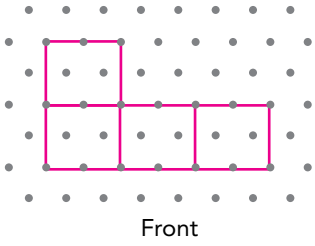
Example 2: Draw the top, the two sides and front views of this solid.

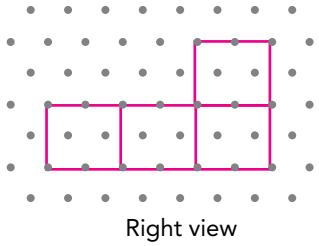


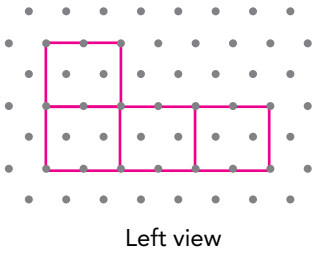
Solution:

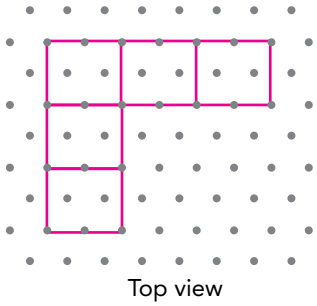


Example 3: Given the various views of a solid, draw a diagram of the solid on isometric paper.

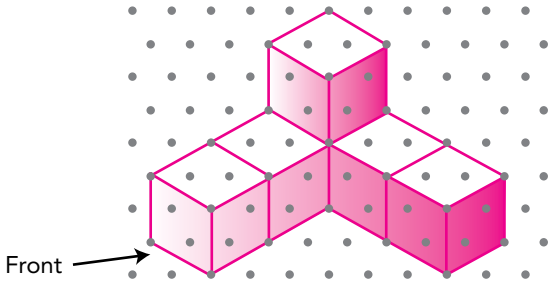








Solution:

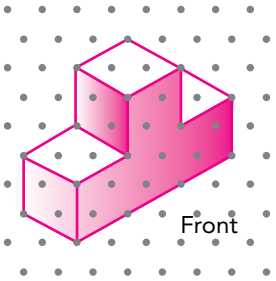


EXERCISE 13.4

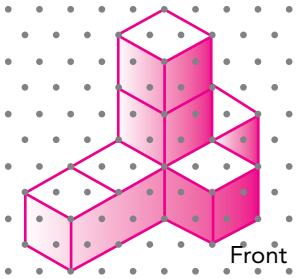
Drawing solids from different perspectives

1. For each of the solids shown, first build the shape using centicubes and then draw the diagrams from the front, top, right and left views.

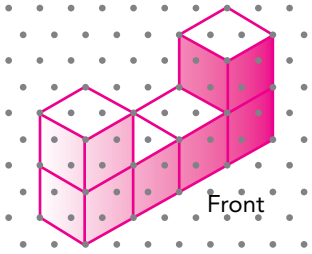
(a)



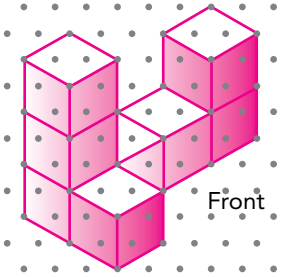
(b)



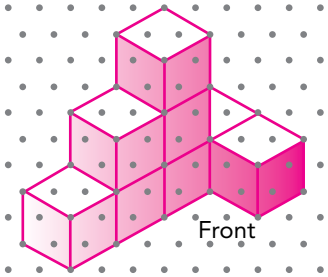
(c)



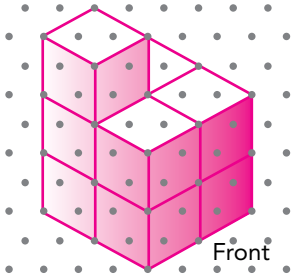
(d)



(e)

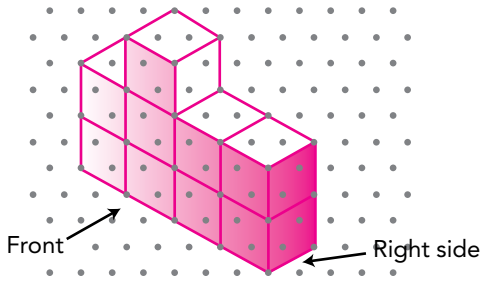


(f)

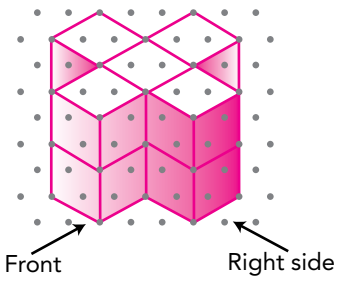


2. Given these solids draw the front, top, right and left views.

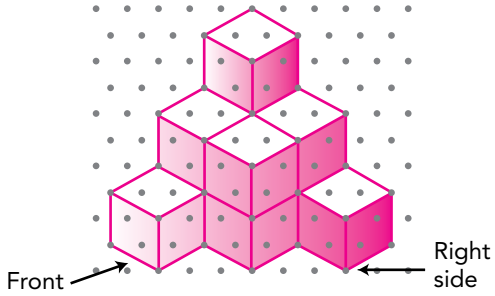
(a)



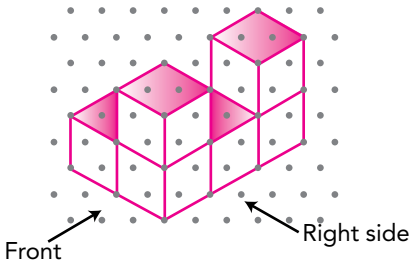
(b)



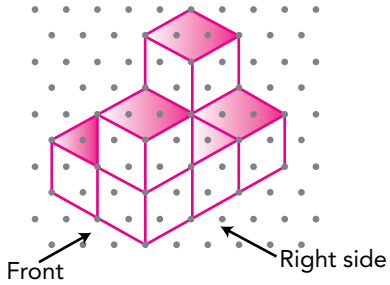
(c)



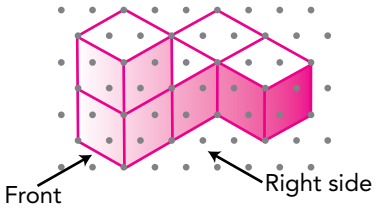
(d)



(e)



(f)



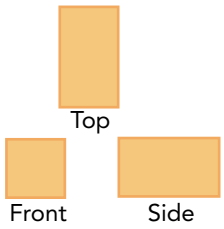
3. Draw the top and side views for:

(a) A cone.

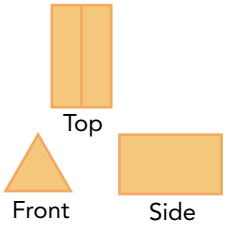
(b) A sphere.

4. Name the solid, given these front, top, and side views.

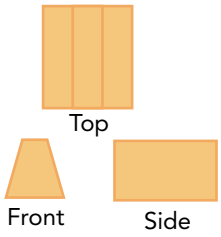
(a)



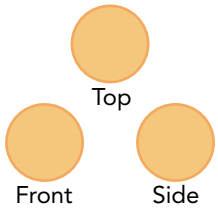
(b)



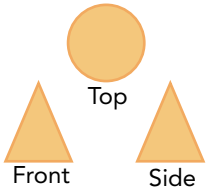
(c)



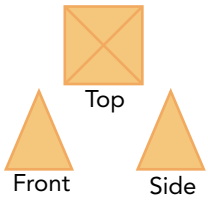
(d)



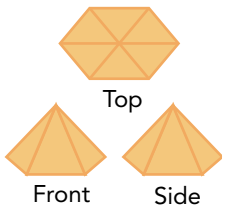
(e)



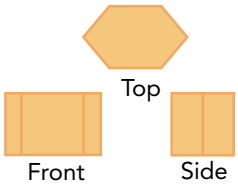
(f)



(g)



(h)



(i)