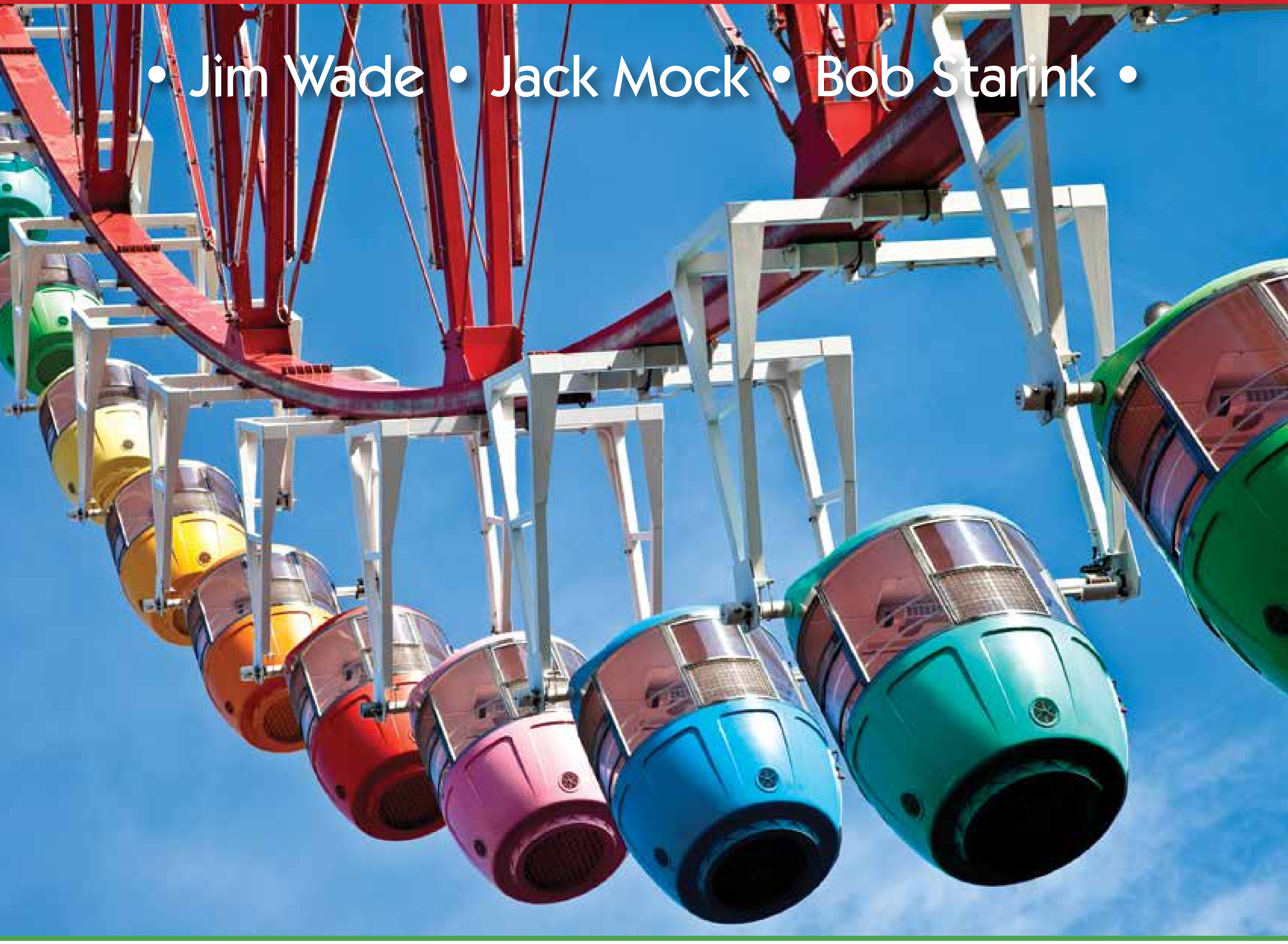
NATIONAL MATHS

YEAR 10



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Chapter 2

Financial Mathematics

Syllabus

Connect the compound interest formula to repeated applications of simple interest using appropriate digital technologies. (ACMNA229)

KEY SKILLS AND KNOWLEDGE

Basic skills (Stage 5.1): By the end of this chapter you should be able to:

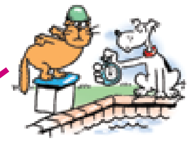
- Calculate compound interest for two or three years using repetition of the formula for simple interest. (2.1)
- Connect the calculation of the total value of a compound interest investment to repeated multiplication using a calculator. (2.2)
- Compare simple interest with compound interest in practical situations. (2.2)
- Compare simple interest with compound interest on an investment over various time periods using tables, graphs or spreadsheets. (2.2)

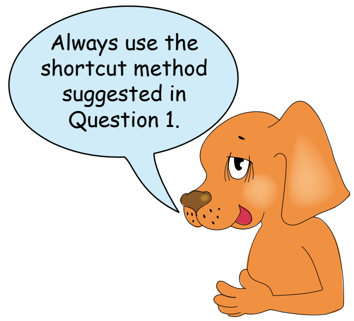
Substantial skills (Stage 5.2): By the end of this chapter you should be able to:

- Establish and use the formula to find compound interest: $A = P(1 + R)^n$. (2.3)
- Calculate and compare investments for different compounding periods. (2.3)
- Use a spreadsheet to graph the value of an investment of a particular amount at various compound interest rates over time. (2.3)
- Solve problems involving compound interest. (2.3)
- Calculate the principal or interest rate needed to obtain a particular total amount for a compound interest investment. (2.3)
- Use a 'guess and refine' strategy to determine the number of time periods required to obtain a particular total amount for a compound interest investment. (2.3)
- Compare the total amounts obtained for a particular investment when the interest is calculated as compound interest and as simple interest. (2.3)
- Use the compound interest formula to calculate depreciation. (2.4)



GETTING STARTED





1. To increase \$40 by 5%, the correct calculation is:

(A) $40 + 0.05$ (B) 40×0.05
(C) 40×1.05 (D) $40 + 1.05$

2. Increase \$60 by 20%.

3. To decrease \$80 by 10%, the correct calculation is:

(A) $80 - 0.1$ (B) $80 - 10$
(C) $80 - 0.9$ (D) 80×0.9

4. Decrease \$25 by 60%.

5. The number of years in 18 months is:

(A) $18 \div 3$ (B) 18×12 (C) $18 \div 12$ (D) $18 \div 30$

6. 0.5% converted to a decimal is:

(A) 0.05 (B) 0.5 (C) 0.005 (D) 0.0005

7. A bill for \$100 has a 10% delivery fee added and then 10% GST is added. What is the value of the final bill?

(A) \$120 (B) \$121 (C) \$122 (D) \$124

8. What is the value of $1.05 \times 1.05 \times 1.05$?

(A) 3×1.05 (B) 3.15 (C) $(1.05)^3$ (D) $\sqrt[3]{1.05}$

9. The value of 1 + 3% is:

(A) 4% (B) 1.03% (C) 1.3 (D) 1.03

10. If a yearly rate of interest is 6% then the rate that will apply for six months of that year is:

(A) 6% (B) 3% (C) 1% (D) 12%

11. How many 6 monthly periods are there in one and a half years?

(A) 15 (B) 2 (C) 3 (D) 9

12. To work out the calculation 3×5^2 in a spreadsheet, the formula is:

(A) = 3 * 5^2 (B) = 3 * 5^2 (C) = 3 × 5^2 (D) = (3 * 5)^2

13. If interest is compounded quarterly it means:

(A) Every 4 months. (B) Every 3 months.
(C) Once every 4 years. (D) Every 4 weeks.

14. Write $0.7\frac{1}{2}\%$ as a decimal.

(A) 0.75 (B) 0.075 (C) 0.0075 (D) 0.00075

15. If a number is multiplied by 0.95 then compared to the original number, the result is:

(A) Larger. (B) Smaller. (C) More than 1. (D) Less than 1.

Section 1: Developing basic skills in applying the simple interest formula successively

2.1 Reviewing simple interest

To calculate the interest earned in the case of an investment (or interest owing in the case of a loan), find the percentage of the principal amount and multiply it by the number of years of the investment (or loan).

Example 1: Calculate the interest earned on investing \$5000 at 3.25% pa for 3 years.

Solution: Calculate the interest earned for 1 year.

3.25% of \$5000 = 0.0325 × 5000 = \$162.50

For 3 years, interest = \$162.50 × 3 = \$487.50

Example 2: An amount of money invested for 8 years at 3.5% pa accrues \$784 in interest. Calculate the amount of the investment.

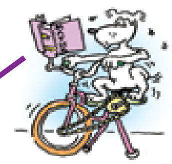
Solution: If \$784 is accrued in 8 years then the yearly interest bill is \$784 ÷ 8 = \$98.

The yearly interest rate is 3.5% so \$98 represents 3.5% of the invested amount.

Using the unitary method 1% = \$98 ÷ 3.5 = \$28

100% = \$28 × 100 = \$2800

EXERCISE 2.1



Reviewing simple interest

1. Calculate the simple interest on these investments.
   (a) \$4000 is invested at 4% pa for 5 years.
   (b) \$2500 is invested at 3.5% pa for 2 years.
   (c) \$500 000 is invested at 2.5% pa for 10 years.
   (d) \$1950 is invested at $3\frac{1}{4}$% pa for 4 years.
   (e) \$20 million is invested at 4.75% pa for 3 years.

2. Use the formula $I = PRN$ where I = simple interest; P = principal, R = rate as a percentage, N = number of years (or time periods), to calculate the interest on these loans.
   (a) \$60 000 is borrowed for 4 years at 3% pa.
   (b) \$200 000 is borrowed for 25 years at 2.5% pa.
   (c) \$8000 is borrowed for a car at 8.5% simple interest over 4 years.
   (d) \$2500 is borrowed for a home theatre system for 3 years at 7.5% pa.
   (e) \$97 500 is borrowed at $6\frac{3}{4}$% pa for 5 years.
   (f) \$50 000 is borrowed at 7% pa for 8 months.
   (g) \$350 is borrowed at 15% pa for 100 days.
   (h) \$12 000 is borrowed at 1.25% per month for 2 years.

3. A loan taken out over 3 years at the rate of 2.5% pa accrued interest charges of \$420.
 (a) How much interest was charged each year?
 (b) If the yearly interest bill is 2.5% of the loan, how much was the loan?

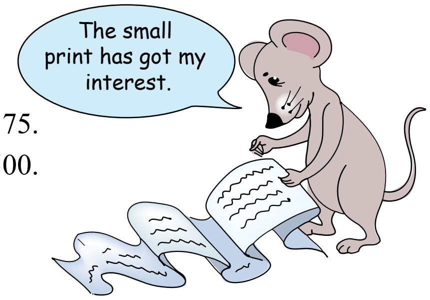
4. Using the concepts broken down in Question 3, calculate the amount of a loan over 5 years taken at 4.5% pa that accrued \$1890 in total interest.

5. Calculate the amount of each loan given the following figures.
 (a) Term = 10 years; rate = 6.5% pa; total interest = \$19 500.
 (b) Term = 15 years; rate = 5.5% pa; total interest = \$9900.

6. To buy a new 4WD Hans takes out a loan of \$48 000 at 3.75% pa simple interest. After several years Hans has paid back the loan plus \$10 800 in interest. How many years was the term of the loan?

7. Calculate the length of the term of each of these loans if the interest is charged at a flat rate.
 (a) Amount = \$64 000; rate = 4.5% pa; total interest = \$34 560.
 (b) Amount = \$250 000; rate = 1.75% pa; total interest = \$78 750.

8. The small print in a car loan contract states that for 5 years interest at a flat rate on a car of \$20 000 value, the total interest paid would be \$7500. What is the interest rate being charged?



9. Calculate the flat interest rate on these loans.
 (a) Amount = \$25 000; term = 6 years; total interest = \$9375.
 (b) Amount = \$12 500; term = 4 years; total interest = \$4500.

10. A bank pays 6.3% pa interest on a term deposit, the interest being credited to your account monthly.
 (a) What is the monthly rate of interest?
 (b) If \$50 000 is invested in the term deposit, how much is credited to your account each month?
 (c) How much interest is earned in 18 months?

11. Investigating compound interest.
In Example 1 above the interest earned on investing \$5000 at 3.25% pa for 3 years was calculated.
The interest earned for 1 year was 3.25% of \$5000 = 0.0325 × 5000 = \$162.50.
If the interest earned is paid yearly and reinvested, how much interest is earned?

Step 1: The interest earned in the first year is \$162.50.
 The principal is now increased by the interest amount = \$5000 + \$162.50 = \$..................
Step 2: Interest earned in the second year is 0.0325 × \$............ = \$................
 The principal is again increased by the interest amount = \$........... + \$.......... = \$.............
Step 3: The interest earned in the third year is 0.0325 × \$.......... = \$........................
 The total interest earned = \$162.50 + \$............... + \$................... = \$503.51
 For 3 years, the amount of flat interest = \$162.50 × 3 = \$487.50
 For 3 years, the amount of compound interest = \$503.51
 Difference = \$503.51 – \$487.50 = \$16.01
 Why is the interest greater when compounded?

12. $4000 is invested at 5.5% pa.

(a) How much interested has been earned at the end of the first year?

(b) If this interest is added to the $4000 principal, how much is available for reinvestment?

(c) If the total is reinvested, how much interest is earned in the second year?

(d) After 2 years, how much is available for reinvestment?

(e) How much interest is earned in the third year?

(f) What is the total (compound) interest earned over 3 years?

(g) Calculate the simple interest earned on a 3-year investment?

(h) What is the difference between the compound interest amount and the simple interest?

13. (a) Calculate the simple interest on an investment of $55 000 at 6.5% pa for 3 years.

(b) Now take the same investment and calculate the interest earned after 1, 2 and 3 years if the interest is added to the principal and reinvested for the second and third years.

(c) How much additional interest is earned by compounding the investment?

14. A bank account containing $8000 earns interest at 2.4% pa. The interest is paid 6 monthly and may be taken as cash (money to live on) or added to the account for reinvestment.

(a) What is the interest rate for 6 months from a yearly rate of 2.4%?

(b) How much cash is paid each 6 months if taken as a living allowance?

(c) How much interested is earned in the second half of the year if the interested is reinvested in the account?

(d) How much extra interest is earned by reinvesting or compounding the interest?

2.2 Finding compound interest by repeated investment

When interest is added to an investment, it is essentially a percentage increase in the principal amount.

To add 5% interest pa is to increase the amount by 5%.

We have already seen that the most efficient way of making a percentage increase is to multiply by 1 plus the percentage increase. In this case multiply by 1.05.

Example 1: $5000 is invested at 5% pa compound interest for 3 years. How much interest is earned?

Solution: In the first year, the amount is increased by 5% giving $5000 × 1.05 = $5250

In the second year, the amount is carried over and increased again by 5% giving $5250 × 1.05 = $5512.50

In the third year, the increase is again applied giving $5512.50 × 1.05 = $5788.13

This is the total amount of the investment.

To calculate the interest gained, the principal must be subtracted.

Interest = $5788.13 – $5000 = $788.13

Example 2: $35 000 is invested into an interest bearing deposit account for 1 year at 6% pa compounded quarterly (i.e. every 3 months). How much will the investment be worth at the end of the year?

Solution: 6% pa is 6 ÷ 4 = 1.5% per quarter.

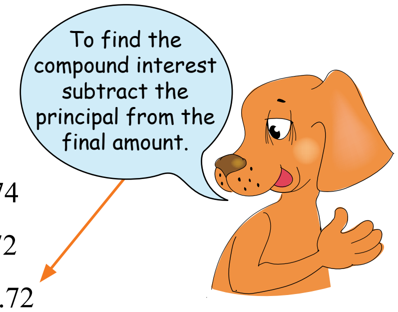
1st quarter increase is $35 000 × 1.015 = $35 525

2nd quarter increase is $35 525 × 1.015 = $36 057.88

3rd quarter increase is $36 057.88 × 1.015 = $36 598.74

4th quarter increase is $36 598.74 × 1.015 = $37 147.72

Note: Interest earned = $37 147.72 – $35 000 = $2147.72

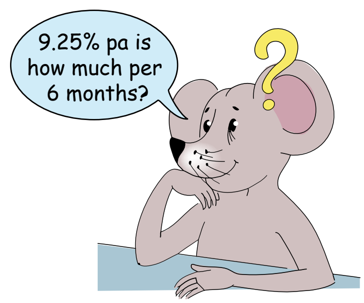


EXERCISE 2.2

Finding compound interest by repeated investment

1. An investment is made in a term deposit paying 6% interest pa.
   (a) By what factor will the principal be multiplied to find the yearly increase?
   (b) If $8000 is invested for a year, use your calculator to find the new balance.
   (c) if $8000 is invested for 2 years find the new balance if the interest is reinvested.
   (d) If $8000 is invested for 3 years, find the new balance if the interest is reinvested and hence find the interest earned.

2. Jason receives $78 000 from an inheritance and puts it into a credit union account, earning $9\frac{1}{4}$% pa interest compounded half-yearly. What will the account be worth after 18 months?



3. $3200 is placed into an account at 2.75% interest compounded annually. How much interest will be earned after 3 years?

4. If $26 000 is invested at 6% and compounded annually for 4 years, how much interest is gained?

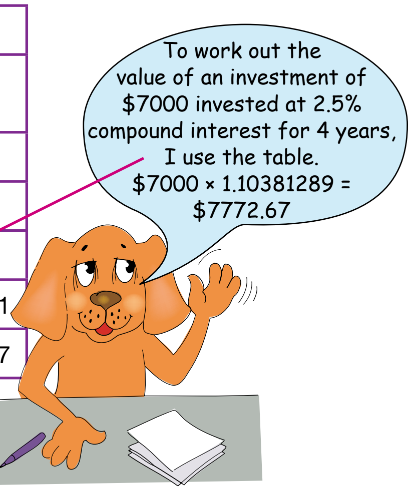
5. Investigation.
   $1000 is invested at an annual interest rate of 4%.
   By what factor will the $1000 be multiplied in order to find the value of the investment after one year?
   If the principal and interest are reinvested at the same rate by what factor will the investment be multiplied to calculate its value for the second year?
   State true or false: The investment will be multiplied by the same growth factor each year.
   Complete:
   The value of the investment after 3 years will be $............... × (1.04) × (1.04) × (1.04) = $...............

6. Using the method discovered in the investigation, calculate the amount to which the given investment will grow.
   (a) $2000 invested at 5% pa compound interest for 3 years.
   (b) $10 000 invested at 3% pa compound interest for 4 years.
   (c) $500 placed in a bank account at 6% pa and interest compounded quarterly for 1 year.

7. A colony of protected penguins numbers 550 and is growing at a rate of 4% annually. How many penguins will be in the colony in 3 years time?

Growth factor table

Rate pa	1%	1.50%	2%	2.50%	3%
Number of years	Growth factor				
1	1.01	1.015	1.02	1.025	1.03
2	1.0201	1.030225	1.0404	1.050625	1.0609
3	1.030301	1.04567838	1.061208	1.0768906	1.092727
4	1.04060401	1.06136355	1.0824322	1.10381289	1.12550881
5	1.05101005	1.077284	1.104080803	1.13140821	1.15927407



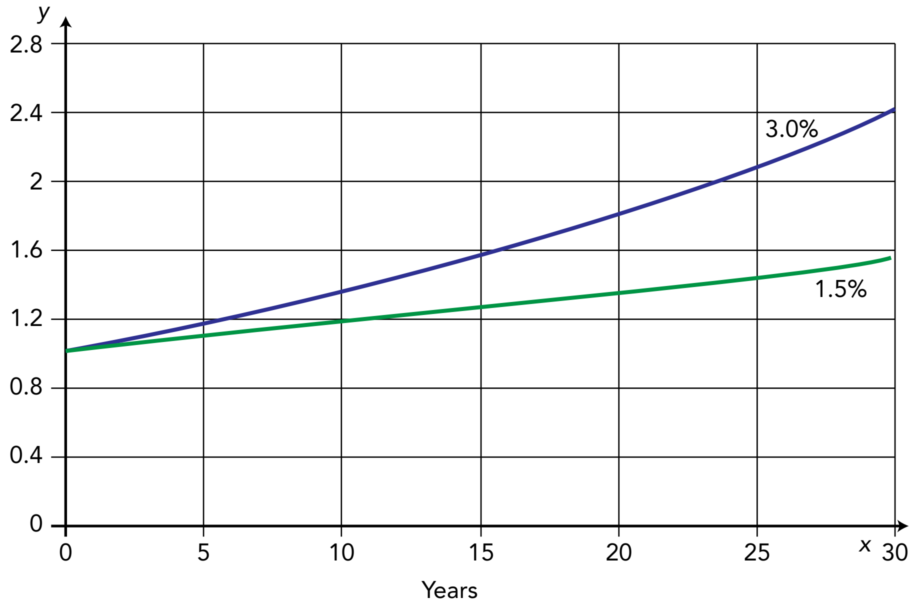
8. Use the growth factor table along with your calculator to work out the value to which these investments will grow. Give your answer correct to the nearest whole cent.
(a) $4500 invested at 2.5% pa for 5 years.
(b) $2000 invested at 3% pa for 3 years.
(c) $10 000 invested at 1% pa per month for 5 months.
(d) $5000 invested at 5% pa for 18 months, interest compounded at 6 monthly intervals. (*Hint*: What rate will be used for a six month interval if the rate is 5% pa?)
(e) What size will a city of 2 000 000 grow to in 5 years if the population growth rate is 1.5% pa?

9. By dividing by the growth factor in the above table, find the required outlay (to the nearest whole number).
(a) How much would I need to invest at 2% pa if I wanted $10 000 in 5 years time?
(b) How much would I need to invest at 2.5% pa if I wanted to buy a car for $45 000 in 3 years time?
(c) If I needed 1000 sheep on a sheep station in 5 years from now, how many sheep would I need to stock if the flock numbers grew by 2% annually?
(d) A wealthy businessman wants to set up a trust for his daughter for when she turns 21. If he wishes her to receive $20 000 on her 21st birthday, how much should he invest for her on her 16th birthday at 3% pa compound interest?

10. Which loan will earn the greater amount of interest? A $10 000 investment at 3% pa simple interest over 5 years or the same investment at 2.5% compound interest?

11. (a) $5000 is invested at 3% compound interest for 2 years. Use the table above to calculate the interest earned.
(b) Another $5000 is invested at 3% compound interest for 4 years. Use the table above to calculate the interest earned.
(c) Does investing for 4 years double the amount of interest obtained over 2 years? Explain.

12. The graphs show the value of \$1 invested at 1.5% pa and 3% pa compound interest respectively over a long period of time.



(a) Read from the graph the value of \$1 invested at 1.5% for 20 years.
(b) Hence find the approximate value of \$1000 invested at 1.5% compound interest for 20 years.
(c) How long does it take a sum of money to double when invested at 3% compound interest?
(d) As time passes, the graphs become further apart. Explain the significance of this for compound interest investment.
(e) State one advantage and one disadvantage of using a graph to find compound interest.

13. Re-create the table above showing the compound interest growth factor by using a spreadsheet. Add extra columns to calculate interest rates up to 5% and extra rows to extend the period of investment to 10 years. Here is a partial spreadsheet showing the formulas used.

Rate pa	**0.01**	**0.015**	**0.02**	**0.025**	**0.03**
Number of years			**Growth factor**		
1	1.01	1.015	1.02	1.025	1.03
2	=(1.01)^2	=(1.015)^2	=(1.02)^2	=(1.025)^2	=(1.03)^2
3	=(1.01)^3	=(1.015)^3	=(1.02)^3	=(1.025)^3	=(1.03)^3
4	=(1.01)^4	=(1.015)^4	=(1.02)^4	=(1.025)^4	=(1.03)^4
5	=(1.01)^5	=(1.015)^5	=(1.02)^5	=(1.025)^5	=(1.03)^5

Use your extended spreadsheet to obtain the growth factors for these investments and hence answer these questions.

(a) From your spreadsheet, find the growth factor for investing \$1 at 4.5% pa for 7 years. Hence find the amount that \$5000 will grow to when invested at 4.5% for 7 years.
(b) Calculate the compound interest earned on investing \$4000 at 3.5% for 8 years.
(c) From your growth table, estimate how long it would take a sum of money to increase by 50% when invested at 5% pa.
(d) Will it take double that amount of time to increase by 100%? Explain.

Section 2 Developing substantial skills in applying the compound interest formula

2.3 Developing and using the compound interest formula

INVESTIGATION

Use the spreadsheet formulas from Question 10 in the previous exercise to complete the following.

\$500 invested at 3% pa for 1 year will grow to \$500 × 1.03 = \$............................ (from calculator)

\$500 invested at 3% pa for 2 years compounded annually will grow to \$500 × Growth factor .... = \$.................

\$500 invested at 3% pa for 3 years compounded annually will grow to \$500 × Growth factor .... = \$.................

\$500 invested at 3% pa for n years compounded annually will grow to \$500 × Growth factor .... = \$.................

Generalising further, $\$P$ invested at 3% pa for n years compounded annually = ………………………

State true or false: The result of investing $\$P$ at 3% pa for n years = $\$P \times (1.03)^n$

Generalising further, $\$P$ invested at r% pa for n years = $\$P \times (1 + R)^n$ where $R = \frac{r}{100}$

$$\boxed{\text{The compound interest formula: } A = P(1 + R)^n}$$

Where A = amount ; P = principal ; R = percentage rate expressed as a decimal ; n = number of years

Example 1: Find the compound interest on \$5000 at 6% pa for 4 years.

Solution:

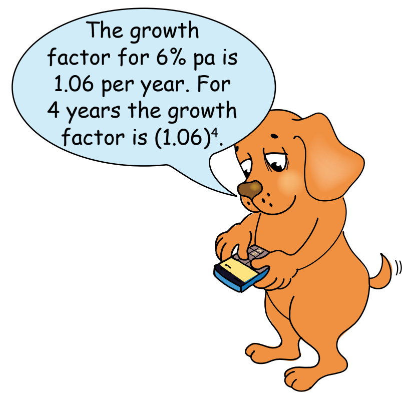
$A = P(1 + R)^n$

$= 5000 \times (1 + 0.06)^4$

$= 5000 \times (1.06)^4$

$= \$6312.38$

Interest = \$6312.38 – \$5000 = \$1312.38



Example 2: Find the compound interest on \$3000 at 4.5% pa for 4 years, interest compounded monthly.

Solution:

4.5% pa = 4.5 ÷ 12 = 0.375% per month.

There are 4 × 12 = 48 months in 4 years.

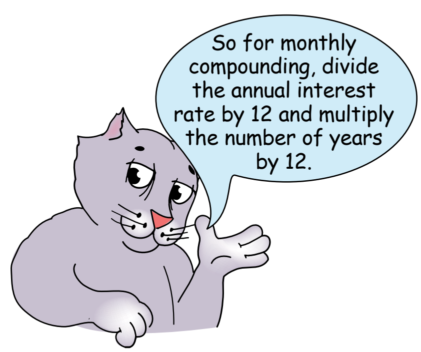
$A = P(1 + R)^n$

$= 3000 \times (1 + 0.00375)^{48}$

$= 3000 \times (1.00375)^{48}$

$= \$3590.45$

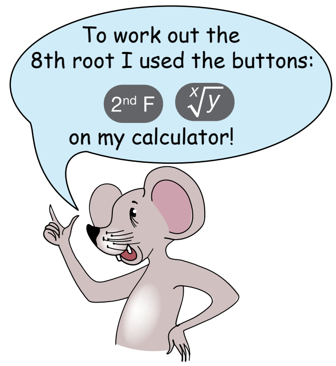
Interest = \$3590.45.38 – \$3000 = \$590.45



Example 3: The value of an investment property increased from \$440 000 to \$750 000 in 8 years. Assuming it increased annually at a constant compound interest rate, find the rate of increase to 1 decimal place.

Solution: $A = 750\,000$, $P = 440\,000$, $n = 8$, $R = ?$

Substitute these values into the compound interest formula $A = P(1 + R)^n$



$750\,000 = 440\,000(1 + R)^8$

$$(1 + R)^8 = \frac{750000}{440000}$$

$$1 + R = \sqrt[8]{\frac{750000}{440000}}$$

$1 + R = 1.06893$

$R = 0.06893 = 6.9\%$ pa

Hence the rate of appreciation is 6.9% pa.

Example 4: The value of a house in a Melbourne suburb appreciates (increases in value) at 8% pa for 5 years. If its current value is \$650 000 what was its original value?

Solution: $A = 650\,000$, $R = 0.08$, $n = 8$, $P = ?$

Substitute these values into the compound interest formula $A = P(1 + R)^n$

$650\,000 = P(1 + 0.08)^5$

$$P = \frac{650000}{(1.08)^5} = \frac{650000}{1.46932}$$

$P = \$442\,381.51$

Example 5: The value of an investment property increased from \$440 000 to \$750 000 in 8 years. Assuming it increased annually at a constant compound interest rate, find the annual rate of increase correct to 1 decimal place.

Solution: $A = 750\,000$, $P = 440\,000$, $n = 8$, $R = ?$

Substitute these values into the compound interest formula $A = P(1 + R)^n$

$750\,000 = 440\,000(1 + R)^8$

$$(1 + R)^8 = \frac{750000}{440000} = 1.7045$$

$1 + R = \sqrt[8]{1.7045}$

$1 + R = 1.0689$

$R = 0.0689$

Hence the rate of appreciation is 6.9% pa.

EXERCISE 2.3

Developing and using the compound interest formula

1. (a) Use the compound interest formula to calculate the final amount after investing $3000 at 4% pa interest compounded annually for 3 years.

(b) How much interest has been earned?

2. Calculate the compound interest earned on these investments by using the formula.

(a) Amount = $40 000; rate = 6.5% pa; term = 10 years.

(b) Amount = $250 000; rate = 3.75% pa; term = 20 years.

(c) Amount = $5000; rate = 9.25% pa; term = 3 years.

(d) Amount = $1500; rate = 5% pa; term = 8 years.

3. Compare these investments.

(a) Which is greater $10 000 invested at 4% pa for 5 years or $10 000 invested at 5% for 4 years?

(b) Which has a greater effect on an investment, the interest rate or the time for which it is invested?

4. (a) If you had two options for a term deposit: investing at 6% pa compound interest for a fixed term of 5 years, or a 6 year investment at 5% compound interest, which would give the greater return?

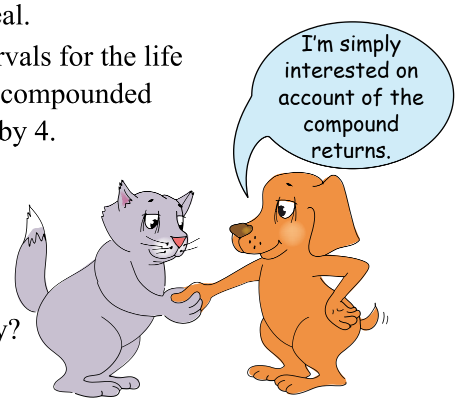
(b) Give two reasons why you might opt for the less lucrative deal.

5. For many investments, the interest is compounded over short intervals for the life of the loan, For example, a 2-year investment might have interest compounded quarterly. The interest rate for a quarter is the annual rate divided by 4.

(a) Calculate the interest earned investing $8000 at 7.5% pa for 3 years, interest compounded annually.

(b) Now calculate the same investment if the interest is compounded quarterly.

(c) How much extra interest is earned by compounding quarterly?



6. $20 000 is invested into an account.

(a) How much interest is earned a year if the return is 8% pa?

(b) How much interest is earned if the interest is compounded monthly?

(c) How much more interest is earned from the compound interest than from the simple interest?

7. Which compounding period would yield the highest amount of interest – daily, monthly, quarterly or yearly?

8. Use the compound interest formula to answer these questions.

On 31 March 2000 the Australian Stock Market index known as the All Ordinaries was restructured to reflect the value of the top 500 Australian companies. At this time the value of the index was 3133.3 points.

(a) If the index had risen at a rate of 6% pa from year 2000 until now (round off to the nearest whole year), calculate the expected value of the index.

(b) Look up the current value of the All Ordinaries index on the internet.

(c) Has the All Ordinaries index increased at a rate greater or less than 6% pa?

(d) What conclusion can you draw about investing in stocks and shares over a long period of time?

9. (a) If China's population was 1 265 830 000 in the year 2000 and it grew at a rate of 1.5%, what would the population be in 2015? (round off the number of years to the nearest year).

(b) Look up the population of China in 2015.

(c) Has the population of China increased at a rate greater or less than 1.5% pa since the year 2000?

(d) From your answer to part (c) estimate China's current growth rate and predict the size of the Chinese population in 10 years time.

10. Create a spreadsheet to calculate the compound interest gained from investing a given sum for a given number of time periods at a given rate of compound interest. The final calculation will require the principal invested to be subtracted from the final amount to find the interest earned.

A sample calculation is shown on row 3, finding the compound interest on an investment of $7000 at 6.25% for 4 years. The formulas used are shown in the columns E and F.

A	B	C	D	E	F
1	**Principal**	**Rate %**	**Number of time periods**	**Final amount**	**Interest**
2	$7000	6.25	4	$8921.01	$1921.01
3				=B4*(1+C4/100)^D4	=E4-B4
4					

Use the spreadsheet you have created to calculate the compound interest earned for these investments.

(a) $6000 invested at 12.25% pa for 7 years.

(b) An endowment fund of $1 000 000 invested for 100 years at 2.5% pa.

(c) $200 invested at 3.75% for 10 years.

(d) $100 invested at 7.5% pa for 3 years, interest compounded monthly.

11. Use the spreadsheet created in Question 10 to analyse this investment. The sum of $10 000 is invested at 5% pa for 1, 2, ... 19, 20 years. How many years is it before the sum is doubled?

12. Use the compound interest formula to calculate how much you would need to invest at 6.5% pa for 10 years, interest compounded annually, in order to achieve a sum of $50 000.

13. How much would you need to invest at 4.5% pa for 15 years, interest compounded annually, in order to have a balance of $20 000.

14. I have an inheritance of $10 000 which has been invested for me at 4% pa compound interest. I want to know how long I should leave it invested if I want it to grow to $15 000.

(a) Use the compound interest formula to calculate the value of my investment after 6 years, 10 years and 14 years.

(b) Between which two time periods will my investment reach $15 000?

(c) Of these two time periods, which gives a value closer to $15 000?

(d) Estimate the number of years I should leave my investment before withdrawing $15 000.

15. Using a trial and error strategy, determine how long it will take $45 000 to grow to $60 000 if it is invested at 4.5% compound interest, compounded annually. Round off your answer to the nearest year.

16. The population in a mining town in Western Australia has been increasing at a rate of 5% pa for the last 3 years. If the population is now 5600, what was the population 3 years ago?

17. The value of a residential property in Brisbane rose from $575 000 to $820 000 in 5 years.
Find the annual (compound) rate of increase.

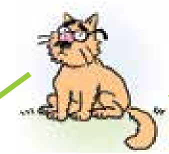
18. The economy is currently running at an inflation rate of about $2\frac{1}{2}$% pa. Find the value of these products in 5 years time if they rise at the rate of inflation.

(a) A litre of milk that now costs $2.40.

(b) A litre of unleaded petrol if it currently costs $1.54.

(c) A new car which now costs $20 000.

WHO AM I?



I was born in England in 1821 and I helped found the British school of pure mathematics. I am known for developing the algebra of matrices and non-Euclidean and n-dimensional geometry. I published many mathematical papers and was the first person to realise that Euclidean geometry was a special case of projective geometry.

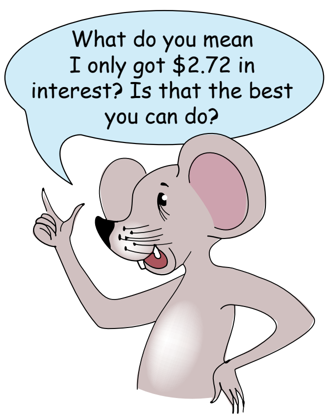
After graduating with honours from Cambridge University with a Master of Arts I was declared Senior Wrangler (dux of Mathematics) and took up a university fellowship there and tutored and wrote many mathematical papers for seven years. I left Cambridge for a profession in law but returned 14 years later as Sadlierian professor of Mathematics. I have a theorem about the characteristic equation of a matrix named after me. My initials are AC.

Who am I?

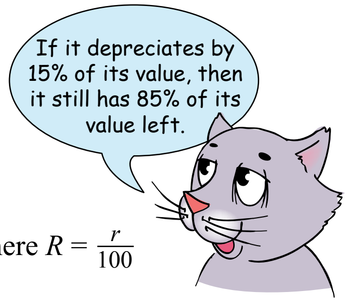
INVESTIGATION

1. How much will \$1 become if invested for 1 year at 100% interest?
2. Manually calculate the amount if the \$1 is invested at 100% pa with interest compounded every 6 months. How much extra interest is earned?
3. If the 100% is compounded monthly, what is the interest rate per month? Use the compound interest formula to find out how much your \$1 will become at 100% pa if the interest is compounded monthly.
4. Continue reducing the compounding time. What is the interest rate per day if 100% is compounded daily? How much will your \$1 become if interest is compounded daily?
5. Is the amount increasing with shorter and shorter time periods? What do you notice about the amount of increase and what does that suggest? Can your \$1 become extremely large?
6. How many hours in 1 year? What is the hourly interest rate for 100% pa? Calculate the amount to which \$1 will grow if invested for 1 year at 100% pa compounded hourly.
7. What is the maximum amount that \$1 will grow to (to the nearest cent) if invested at 100% pa interest compounded over an infinite number of time periods?
8. From your calculator, use the e^x function to find the value of $e^1 = e$ (the exponential growth constant).
9. State true or false: $e = \underset{n\to\infty}{Lim}\left(1+\frac{1}{n}\right)^n$

   The irrational number 'e' was described by Leonhard Euler and is called Euler's number.



2.4 Depreciation



Depreciation is similar to compound interest except that the value of the investment or asset *decreases* each year by a fixed percentage. The growth factor is less than 1 and is found by subtracting the depreciation rate from 1.

An asset initially worth $\$P$ depreciates at r% pa for n years = $\$P \times (1 - R)^n$ where $R = \frac{r}{100}$

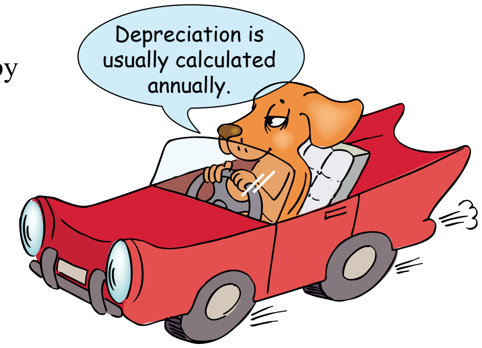
The depreciation formula: $V = P(1 - R)^n$

Where V = value; P = principal value; R = percentage depreciation rate expressed as a decimal; n = number of years.

Example 1: A motor vehicle is insured for \$35 000 when new and depreciates by 15% pa.

What is the insured value of the car after 3 years?

Solution: Note that the growth factor is < 1. If it depreciates by 15%, then its value is 1 – 0.15 = 0.85 or 85% of the previous year.



$V = P(1 - R)^n$

$= 35\,000 \times (1 - 0.15)^3$

$= 35\,000 \times (0.85)^3$

= \$21 494 (nearest dollar).

Example 2: A library has depreciated at a rate of $12\frac{1}{2}$% pa for the past 8 years. If the value of the books is now \$20 000, what was the original value of the library?

Solution: $V = 20\,000$, $R = 12\frac{1}{2}\% = 0.125$, $n = 8$, $P = ?$

Substitute theses values in the depreciation formula $V = P(1 - R)^n$

$20\,000 = P(1 - 0.125)^8 = P(0.875)^8$

$P = \frac{20000}{(0.875)^8} = \$58\,205.71$

Hence the original value of the library was \$58 205.71.

Example 3: Helga's notebook was purchased for \$3500. After it depreciated at a constant rate for 5 years it is now worth \$1000. Find its rate of depreciation (correct to 1 decimal place).

Solution: $V = 1000$, $P = 3500$, $n = 5$, $r = ?$

Substitute these values into $V = P(1 - R)^n$

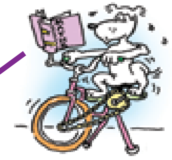
$1000 = 3500(1 - R)^5$

$(1 - R)^5 = \frac{1000}{3500}$

$1 - R = \sqrt[5]{\frac{1000}{3500}} = 0.779$

$R = 1 - 0.779 = 0.221 = 22.1\%$ pa

EXERCISE 2.4



Depreciation

1. Calculate the depreciated value of the following.

(a) A \$1200 video camera depreciating at 15% pa after 3 years.

(b) A \$23 000 car depreciating at 16% pa after 4 years.

(c) A \$3200 notebook computer depreciating at $17\frac{1}{2}$% pa after 5 years.

(d) A \$39 500 car depreciating at 15% for the first year and then $12\frac{1}{2}$% for the next 2 years.

(e) A \$2650 fridge depreciating at 14% for the first 2 years and then $7\frac{1}{2}$% for the next 3 years.

2. Calculate these depreciated values and find the amount of depreciation that may be claimed as a tax deduction.

(a) An accountant is able to claim that the business owns a professional library valued at \$18 000 and depreciates at 15% pa. What is the book value of the library after 3 years?

(b) What depreciation (loss) may be claimed on the library over 3 years?

(c) The office furniture and equipment at a dental surgery is valued at \$66 000 but must be replaced periodically. The dental practice depreciates its value at 18% pa. What is the value of the office after 4 years?

(d) What depreciation (loss) may be claimed on the dental fittings over 4 years?

(e) A truck has an initial value of \$80 000 but depreciates at 20% pa. Find its value after 5 years.

(f) What depreciation (loss) may be claimed by the truck owner after 5 years?

(g) Your accountant suggests that your laptop computer can be depreciated at 25% pa and the lost value claimed as a tax deduction. The initial value of the laptop is \$2464. What is the book value of the laptop after 4 years?

(h) What depreciation (loss) may be claimed on the laptop over 4 years?

3. In Questions 2 (e) and (g) above, the truck is depreciated at 20% for 5 years and the computer is depreciated at 25% pa for 4 years. Since 5 × 20% = 100% and 4 × 25% = 100%, it may appear that the items have lost all of their value and are worth nothing. Explain why this is not so.

4. A \$1500 television set depreciates at 20% pa.

(a) What is the value of the TV after 5 years?

(b) If the value after 5 years was calculated using *straight line depreciation* (equivalent of simple interest), what would the TV be valued at after 5 years?

(c) Using normal depreciation, when will the value of the TV be zero? Why?

5. In a certain country, the birth rate has fallen below the death rate leading to a shrinking of the population. Currently there are 25 000 000 people residing in the country but the population is declining at 5% pa. What is the projected population of the country in 10 years from now?

6. Grandma and grandpa's superannuation investment totals \$400 000 but the law requires that they withdraw 10% of the super fund each year. Grandpa says that's not too bad because the fund is earning 6% pa interest meaning the deficit is just 4% pa. How much money will be in the super fund in 20 years time?

7. A rare bottle of wine is valued initially at \$300.

Because of its rarity, the wine is somewhat of a collector's item and for the first 10 years of its life it appreciates (increases in value) at 5% pa. After 10 years, the concern is that the wine, although of excellent quality may be spoilt when opened due to air leaking into the bottle so the wine begins to depreciate at 5% pa.

(a) What is the value of the wine after the first 10 years?

(b) What is the value of the wine after the second 10 years?

(c) Explain why the wine is not worth \$300 after 20 years.

8. The assay (geological report) of a gold mine reports that gold can be extracted from the mine at a rate of 47 grams per tonne. The mine operator knows that as the richest veins of gold are extracted, the yield from the gold mine will decrease at a rate of about 12% pa. The operator also knows that the mining operation will become unviable when the rate of extraction falls below 15 grams per tonne.

(a) Calculate the expected yield of gold per tonne after 5 years.

(b) Calculate the expected yield of gold per tonne after 10 years.

(c) Use a trial and error method to determine the expected working life of the mine.

9. Stephen bought a new car 6 years ago. The car is currently valued at \$18 400.

It has been depreciating at a rate of $7\frac{1}{2}$% pa for the past 6 years. What was the original value of the car?

10. A teacher's professional library is now worth \$2500 but has been depreciating at the rate of 10% pa for the last 10 years. What was the value 5 years ago?

11. (a) Nina bought a new computer 3 years ago for \$3400. After 5 years it is now worth only \$800. Find its annual rate of depreciation over this time period.

(b) A new tractor cost \$44 800 8 years ago. If after 8 years it is worth \$18 500, find the annual rate of depreciation.

12. A certain country town had a falling population. Currently it has 1375 people and is falling at the rate of $7\frac{1}{2}$% pa. Estimate when the population will first drop below 1000.

EXERCISE 2.5

Miscellaneous extension exercise

1. On your calculator, the symbol '!' means factorial. It is a function that multiplies successive integers down to 1. For example, $5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$.

(a) Using your calculator, find the sum of $1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \frac{1}{5!} + \frac{1}{6!}$

(b) Does the *value* of this series look familiar?

(c) Extend the series to 10 or more terms either on your calculator or by making a spreadsheet (formulas shown below). Write down the result correct to 4 decimal places. What result have you found?

	A	B	C
1	**n**	**n!**	**Sum**
2	0	=FACT(A2)	=B2
3	=A2+1	=FACT(A3)	=C2+1/B3
4	=A3+1	=FACT(A4)	=C3+1/B4

2. (a) If a perspex panel cuts out 15% of the light passing through it, what percentage of light passes through it?
(b) What percentage of light would pass through three perspex panels placed together?
(c) What percentage of light passes through four perspex panels.
(d) How many perspex panels are required to cut out at least 50% of the light?

3. In a certain country, the birth rate is found to be 4% pa. The death rate is known to be 2.5% pa. If there are currently 25 000 000 people living in the country, what is the expected population in 20 years time?

4. Angelina bought a new smartphone 3 years ago for $960. After 3 years it is now worth only $200. Find the yearly rate of depreciation over the 3 years.

5. When repaying a home loan, the interest scheme is usually 'reducible' interest. This means that you only pay interest on how much is still owing. As the loan is gradually paid off, the interest becomes less and more of the monthly payment goes towards reducing the loan and less towards paying the interest.

A flat interest rate keeps charging the same amount of interest based on the original loan, no matter how small the amount still owing.

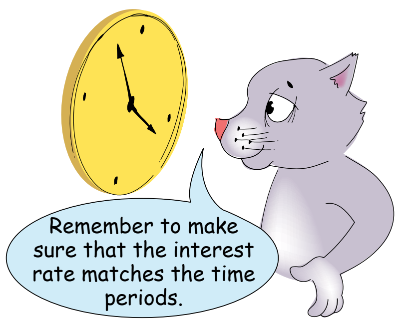
The formula $E = \frac{(1+r)^n - 1}{n}$ converts the flat simple interest rate to the 'effective' reducible rate.
(a) Calculate the annual reducible interest rate (effective interest rate) if $450 000 is borrowed at 6% simple interest, with payments being made monthly over 20 years.
(b) Calculate the annual reducible interest rate (effective interest rate) if $25 000 is borrowed at 4% simple interest, with payments being made quarterly over 10 years.

6. The local credit union advised Henri that a loan of $5000 for a holiday would cost him $111.21 per month over 5 years.
(a) How much money did he repay altogether?
(b) What was the total interest that he paid?
(c) What would be the yearly interest charged if the same amount is charged each year?
(d) Find the flat interest rate charged.
(e) Calculate the equivalent reducible (effective) rate of interest.

7. If a regular payment is made into a savings account or a superannuation fund, with compound interest applied to the account, there is a formula which can tell us the future value of that account.

The future value is $FV = M\left(\frac{(1+R)^n - 1}{R}\right)$ where M is the amount deposited at regular intervals.

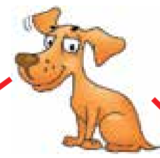
R is the compound interest rate expressed as a decimal and n = number of time periods.
(a) Calculate the future value of an investment where $600 is deposited monthly at 1.5% pa for 30 months.
(b) Calculate the future value of an investment where $500 is deposited quarterly at 8% pa for 10 years.
(c) Calculate the future value of an investment where $400 is deposited monthly at 7.2% pa for 15 years.



8. Another more complicated formula can be used to determine the monthly repayments of a loan if the interest is reducible.

$M = \frac{Ar(1+r)^n}{(1+r)^n - 1}$, where A is the amount borrowed, r is the interest rate per time period and n is the number of time periods.
(a) Use your calculator skills to determine the monthly repayments for a loan where $5000 is borrowed at 6% per annum compounded monthly for 2 years.
(b) Multiply the monthly repayments from part (a) by the number of months and hence calculate the amount of interest charged on the loan.

HOW MUCH DO YOU KNOW?



You are nearly at the end of the chapter. Check that you are able to do the following.

Calculate simple interest (2.1)

- Use the formula $I = PRN$

  Example: Calculate the simple interest on \$5000 invested at 3% pa for 6 years.

  Solution: $I = PRN = 5000 \times 0.03 \times 6 = \$900$

- Reverse use of the formula $I = PRN$ to find P, R or N.

  Example 1: \$500 invested for 2 years in a term deposit earned \$55 in interest. Find the rate.

  Solution: $I = PRN$

  $55 = 500 \times R \times 2$

  $55 = 1000 \times R$

  $R = 0.055$. The rate is 5.5% pa.

  Example 2: \$250 000 is invested at 6.25% pa simple interest. If the investment earned \$125 000 in interest, what was the term of the investment?

  Solution: $I = PRN$

  $125\,000 = 250\,000 \times 0.0625 \times N$

  $125\,000 = 15625 \times N$

  $N = 125\,000 \div 15625 = 8$. The term of the investment was 8 years.

- Cases where the investment does not run for a whole number of years.

  Example: \$500 is invested at 8% pa for 7 months. How much interest is earned?

  Solution: $I = PRN$

  $= 500 \times 0.08 \times \frac{7}{12}$

  $= 23.\dot{3}$. The interest earned was \$23.33.

Find the compound interest by repeated investment (2.2)

- The interest earned each year is reinvested with the principal to earn compound interest.
- Find the annual growth factor. The amount invested increases by the interest rate.

  If the principal is invested at 5% pa then it increases in the ratio 1.05. Amount $= P \times 1.05$.

  For an investment of say 3 years, the amount will increase each year by the same ratio.

  After 3 years the amount $= P \times 1.05 \times 1.05 \times 1.05$.

  Example: \$10 000 is invested at 4% pa for 2 years. Find the compound interest earned.

  Solution: The growth factor for each year is $1 + 0.04 = 1.04$.

  After 2 years, the amount $= \$10\,000 \times 1.04 \times 1.04 = \$10\,816$.

  The interest is found by subtracting the principal. $\$10\,816 - \$10\,000 = \$816$.

- The interest may be compounded at smaller intervals than a year. In these cases the interest rate must be adjusted to match the time interval.
- Example: \$4000 is invested for 2 years at 6% p.a., the interest being compounded 6 monthly. How much does it grow to?

  Solution: First work out the interest rate for 6 months. 6% pa = 3% for 6 months.

  There are 4 lots of 6 months in 2 years. \$4000 × 1.03 × 1.03 × 1.03 × 1.03 = \$4370.91.

Develop and use the compound interest formula (2.3)

- From earlier calculations involving repeated investment: $A = P(1 + R)^n$

  Where A = amount; P = principal, R = percentage interest rate = $\frac{r}{100}$; n = number of time periods.

  Note: The interest rate must be quoted to match the time periods.
- Calculate compound interest using the formula.

  Example: Find the compound interest on \$3000 at 4% pa for 5 years.

  Solution: $A = P(1 + R)^n$

  $= 3000 \times (1 + 0.04)^5$

  $= 3000 \times (1.04)^5 = \$3649.96$

  Interest = \$3649.96 – \$3000 = \$649.96
- Calculate interest compounded over smaller time periods.

  Example: Find the compound interest on \$2000 at 3.6% pa for 3 years, interest compounded monthly.

  Solution: 3.6% pa = 3.6 ÷ 12 = 0.3% per month.

  There are 3 × 12 = 36 months in 3 years.

  $A = P(1 + R)^n$

  $= 2000 \times (1 + 0.003)^{36}$

  $= 2000 \times (1.003)^{36} = \$2227.74$

  Interest = \$2227.74 – \$2000 = \$227.74
- Find the interest rate given the initial and final values and the length of time.

  Example: A bank account grows from \$1200 to \$1400 in 5 years. Assuming it increased annually at a constant compound interest rate, find the rate of increase to 1 decimal place.

  Solution: $A = 1400$, $P = 1200$, $n = 5$, $R = ?$

  Substitute these values into the compound interest formula $A = P(1 + R)^n$

  $1400 = 1200(1 + R)^5$

  $(1 + R)^5 = \frac{1400}{1200}$ (To work out the 5th root use the calculator buttons 2ndF $\sqrt[x]{y}$)

  $1 + R = \sqrt[5]{\frac{1400}{1200}} = \sqrt[5]{1.16666}$

  $1 + R = 1.0313$

  $R = 0.0313 = 3.1\%$ pa

Calculate depreciation (2.4)

- Some items lose value (depreciate) as they get older. Depreciation formula is $V = P(1 - R)^n$.

  Example 1: A caravan is insured for \$45 000 when new and depreciates by 18% pa. What is the insured value of the caravan after 4 years?

  Solution: Note that the growth factor is < 1. If it depreciates by 18%, then its value is $1 - 0.18 = 0.82$

  $V = P(1 - R)^n$

  $= 45\,000 \times (1 - 0.18)^4$

  $= 45\,000 \times (0.82)^4$

  = \$20 345 (nearest dollar).

  Example 2: Joseph bought a rental property and had the contents valued at \$35 000. The furniture and fittings depreciated at a constant rate for 3 years are now worth \$25 000. Find the rate of depreciation (correct to 1 decimal place).

  Solution: $V = 25\,000$, $P = 35\,000$, $n = 3$, $r = ?$

  Substitute these values into $V = P(1 - R)^n$

  $25\,000 = 35\,000(1 - R)^3$

  $(1 - R)^3 = \dfrac{25000}{35000}$

  $1 - R = \sqrt[3]{\dfrac{25000}{35000}} = 0.8939$

  $R = 1 - 0.8939 = 0.1061 = 10.6\%$ pa.

  The rate of depreciation was 10.6% pa.

CHAPTER 2 DIAGNOSTIC TEST

1. Calculate the simple interest earned on \$13 980 invested at 5.1% pa for $4\frac{1}{2}$ years. **2.1**

2. \$2000 is invested at 4.5% pa. **2.1**

(a) How much interest has been earned at the end of the first year?
(b) If this interest is added to the \$2000 principal, how much is available for reinvestment?
(c) If the total is reinvested, how much interest is earned in the second year?
(d) After 2 years, how much is available for reinvestment?
(e) How much interest is earned in the third year?
(f) What is the total (compound) interest earned over 3 years?
(g) Calculate the simple interest earned on a 3-year investment?
(h) What is the difference between the compound interest amount and the simple interest?

3. (a) By what factor will you multiply \$4000 in order to increase it by 5%? **2.2**
(b) \$4000 is invested at 5% interest. What is the value of the investment after 1 year?
(c) If the principal and interest is reinvested what is the amount after 2 years?
(d) What is the growth factor that will increase the amount by 5% annually.
(e) Write an expression for the amount after \$4000 is invested at 5% compound interest for 3 years.
(f) Calculate the amount after \$4000 is invested at 5% compound interest for 3 years.
(g) How much interest has been earned on the \$4000 over the 3 years?

4. Use the growth factor table to calculate the required amounts. **2.2**

(a) Calculate the amount to which \$2000 will grow if invested at $2\frac{1}{2}$% compound interest for 4 years.
(b) Calculate the amount to which \$3500 will grow if invested at 3% pa for 5 years.
(c) How much compound interest will \$50 000 earn if invested at 2% compound interest for 4 years?

Rate pa	1%	1.50%	2%	2.50%	3%
Number of years	Growth factor				
1	1.01	1.015	1.02	1.025	1.03
2	1.0201	1.030225	1.0404	1.050625	1.0609
3	1.030301	1.04567838	1.061208	1.0768906	1.092727
4	1.04060401	1.06136355	1.0824322	1.10381289	1.12550881
5	1.05101005	1.077284	1.104080803	1.13140821	1.15927407

5. The formula for calculating compound interest is: **2.3**

(A) $A = PRN$
(B) $A = P(1 - R)^n$
(C) $A = P(1 + R)^n$
(D) $I = PR^n$

6. (a) What is the value of an account where \$5800 is invested at 2.8% per annum with compound interest compounded quarterly for 2 years? **2.3**

(b) Sam invested \$3650 into an investment fund which paid compound interest at 6.5% per half year. Find the amount of the fund after 4 years.

7. How much interest is earned on an account with a balance of \$35 000 paying 4.8% pa interest compounded monthly for 5 years? **2.3**

8. The value of an investment property has increased at 8% pa for the past 6 years. It is now worth \$485 000. What was its value 6 years ago? **2.3**

9. The value of gold was US\$325 an ounce 4 years ago. It is now US\$415 an ounce, find its rate of appreciation if we assume that it compounded annually. **2.3**

10. The population of a city is 2.3 million and growing at 2% pa. Estimate the population in 10 years time. **2.3**

11. A bank account contains \$500. Interest is paid at 2.4% pa compounded monthly. How much will the bank account contain after 1 year? **2.3**

12. The formula for calculating depreciation is: **2.4**

(A) $V = ARN$

(B) $V = A(1 - R)^n$

(C) $V = A(1 + R)^n$

(D) $V = AR^n$

13. What is the depreciated value of a motorbike bought for \$18 000 after 3 years depreciating at $27\frac{1}{2}$% per annum? **2.4**

14. How much value has a \$2400 computer lost after 4 years if the depreciation rate is 35% pa? **2.4**

15. (a) A professional library has been depreciating at the rate of 20% pa for 6 years. If it is now worth \$5600 what was its original value? **2.4**

(b) Anna bought a new Holden for \$37 000 and its book value is now \$22 500 after 5 years. Find its rate of depreciation.

Chapter 2 Financial Mathematics

Getting started

1 C **2** $72 **3** D **4** $10 **5** C **6** C **7** B **8** C **9** D **10** B **11** C **12** A **13** B **14** C **15** B

2.1 Reviewing simple interest

1 (a) $800 (b) $175 (c) $125 000 (d) $562.50 (e) $2 850 000 **2** (a) $7200 (b) $125 000 (c) $2720 (d) $5625 (e) $32 906.25 (f) $2333.33 (g) $14.38 (h) $300 **3** (a) $140 (b) $5600 **4** $8400 **5** (a) $30 000 (b) $12 000 **6** 6 years **7** (a) 12 years (b) 18 years **8** 7.5% **9** (a) 6.25% (b) 9% **10** (a) 0.525% (b) $262.50 (c) $4725 **11** Step 1: $5162.50. Step 2: 0.0325 × $5162.50 = $167.78. Amount = $5162.50 + $167.78 = $5330.28. Step 3: 0.0325 × $5330.28 = $173.23. The total interest earned = $162.50 + $167.78 + $173.23 = $503.51. Interest is greater when compound because interest is earned on the interest. **12** (a) $220 (b) $4220 (c) $232.10 (d) $4452.10 (e) $244.87 (f) $696.97 (g) $660 (h) $36.97 **13** (a) $10 725 (b) $11 437.23 (c) $712.23 **14** (a) 1.2% (b) $96 (c) $97.15 (d) $1.15

2.2 Finding compound interest by repeated investment

1 (a) 1.06 (b) $8480 (c) $8988.80 (d) $9528.13, $1528.13 **2** $89 330.76 **3** $271.33 **4** $6824.40 **5** $1000 × (1.04) × (1.04) × (1.04) = $1124.86 **6** (a) $2315.25 (b) $11 255.09 (c) $530.68 **7** 619 **8** (a) $5091.34 (b) $2185.45 (c) $10 510.10 (d) 5384.45 (e) $2 154 568 **9** (a) $9057 (b) $41 787 (c) 906 (d) $17 252 **10** 3% × 5 = 0.15. From growth table 3%, 5Y = 1.15927 (best). **11** (a) $ 304.50 (b) $627.54 (c) No. As the investment earns interest the capital increases earning even more interest. **12** (a) $1.35 (b) $1350 (c) 23.5 years (d) As interest is earned, the capital increases earning more interest. (e) Advantage: Quick look-up, disadvantage: Not accurate. **13** (a) 1.36086, $6804.31 (b) $1267.24 (c) About 8 years (d) No. Seethe graphs in Question 11 – compound interest increases at an increasing rate.

2.3 Developing and using the compound interest formula

1 (a) $3374.59 (b) $374.59 **2** (a) $35 085.50 (b) $272 038 (c) $1519.80 (d) $716.18 **3** (a) $10 000 invested at 4% pa for 5 years (b) Time. **4** (a) 6 years at 5% (b) Do not want to tie your money up for 6 years. Better deals may come up in 5 years time. 5 (a) $1938.38 (b) $1997.73 (c) $59.35 **6** (a) $1600 (b) $1659.99 (c) $59.99 **7** Daily **8** (a) Your year data (b) Look up All Ordinaries (c) Varies (d) Returns increase **9** (a) 1 582 581 257 (b) 1 369 455 447 (c) No (d) 1.4%, 2 027 128 599 **10** (a) $7472.73 (b) $10 813 716.35 (c) $89.01 (d) $25.14 **11** 15 years **12** $26 636 **13** $10 334.41 **14** (a) $12 653.19, $14 802.44, $17 316 (b) Between 10 and 14 years. (c) 10 years (d) 11 years. **15** 7 years **16** 4837 **17** 7.4% **18** (a) $2.72 (b) $1.74 (c) $22 628

Who am I? Arthur Cayley.

Investigation

1 $2 **2** $2.25, 25 cents **3** $8.\dot{3}$%, $2.61 **4** 0.274%, $2.71 **5** Yes, but by a smaller amount each time. No **6** 8760, 0.0114%, $2.72 **7** $2.72 **8** 2.7183 **9** True.

2.4 Depreciation

1 (a) $736.95 (b) $11 451 (c) $1223 (d) $25 706 (e) $1551 **2** (a) $11 054 (b) $6946 (c) $29 840 (d) $36 160 (e) $26 214 (f) $53 786 (g) $780 (h) $1684 **3** As the item loses value it depreciates by less each year. **4** (a) $492 (b) 0 (c) Never. It is always worth 80% of the previous year's value. **5** 14 968 423 **6** $176 800 **7** (a) $489 (b) $293 (c) It is depreciating from a larger value and loses more each year than it previously gained. **8** (a) 24.8 grams per tonne (b) 13.1 grams per tonne (c) 8 to 9 years **9** $29 375 **10** $7170 **11** (a) 25% (b) 10.5% **12** In about 4 years.

2.5 Miscellaneous extension exercise

1 (a) 2.7181 (b) Looks similar to value for *e*. (c) 2.7183. The series generates the value of *e*. **2** (a) 85% (b) 61% (c) 52% (d) 5 **3** 33 671 375 **4** 40% **5** (a) 11.6% pa (b) 4.9% **6** (a) $6672.60 (b) $1672.60 (c) $334.52 (d) 6.7% pa (e) 7.9% pa **7** (a) $18 330.09 (b) $30 200.99 (c) $129 012.81 **8** (a) $221.60 (b) $5318.47, Interest = $318.47

Chapter 2 Diagnostic test

1 $3208.41 **2** (a) $90 (b) $2090 (c) $94.05 (d) $2184.05 (e) $98.28 (f) $282.33 (g) $270 (h) $12.33 **3** (a) 1.05 (b) $4200 (c) $4410 (d) 1.05 (e) $4000 \times (1.05)^3$ (f) $4630.50 (g) $630.50 **4** (a) $2207.63 (b) $4057.46 (c) $4121.61 **5** C **6** (a) $6132.87 (b) $6040.73 **7** $9472.43 **8** $305 632 **9** 6.3% **10** 2 803 687 **11** $512.13 **12** B **13** $6859 **14** $1971.59 **15** (a) $21 362 (b) 10.5%