Mastering Physics

MODULE 5 NSW

Advanced Mechanics

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Contents

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Words to Watch

account, account for State reasons for, report on, give an account of, narrate a series of events or transactions.

analyse Interpret data to reach conclusions.

annotate Add brief notes to a diagram or graph.

apply Put to use in a particular situation.

assess Make a judgement about the value of something.

calculate Find a numerical answer.

clarify Make clear or plain.

classify Arrange into classes, groups or categories.

comment Give a judgement based on a given statement or result of a calculation.

compare Estimate, measure or note how things are similar or different.

construct Represent or develop in graphical form.

contrast Show how things are different or opposite.

create Originate or bring into existence.

deduce Reach a conclusion from given information.

define Give the precise meaning of a word, phrase or physical quantity.

demonstrate Show by example.

derive Manipulate a mathematical relationship(s) to give a new equation or relationship.

describe Give a detailed account.

design Produce a plan, simulation or model.

determine Find the only possible answer.

discuss Talk or write about a topic, taking into account different issues or ideas.

distinguish Give differences between two or more different items.

draw Represent by means of pencil lines.

estimate Find an approximate value for an unknown quantity.

evaluate Assess the implications and limitations.

examine Inquire into.

explain Make something clear or easy to understand.

extract Choose relevant and/or appropriate details.

extrapolate Infer from what is known.

hypothesise Suggest an explanation for a group of facts or phenomena.

identify Recognise and name.

interpret Draw meaning from.

investigate Plan, inquire into and draw conclusions about.

justify Support an argument or conclusion.

label Add labels to a diagram.

list Give a sequence of names or other brief answers.

measure Find a value for a quantity.

outline Give a brief account or summary.

plan Use strategies to develop a series of steps or processes.

predict Give an expected result.

propose Put forward a plan or suggestion for consideration or action.

recall Present remembered ideas, facts or experiences.

relate Tell or report about happenings, events or circumstances.

represent Use words, images or symbols to convey meaning.

select Choose in preference to another or others.

sequence Arrange in order.

show Give the steps in a calculation or derivation.

sketch Make a quick, rough drawing of something.

solve Work out the answer to a problem.

state Give a specific name, value or other brief answer.

suggest Put forward an idea for consideration.

summarise Give a brief statement of the main points.

synthesise Combine various elements to make a whole.

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Projectile Motion

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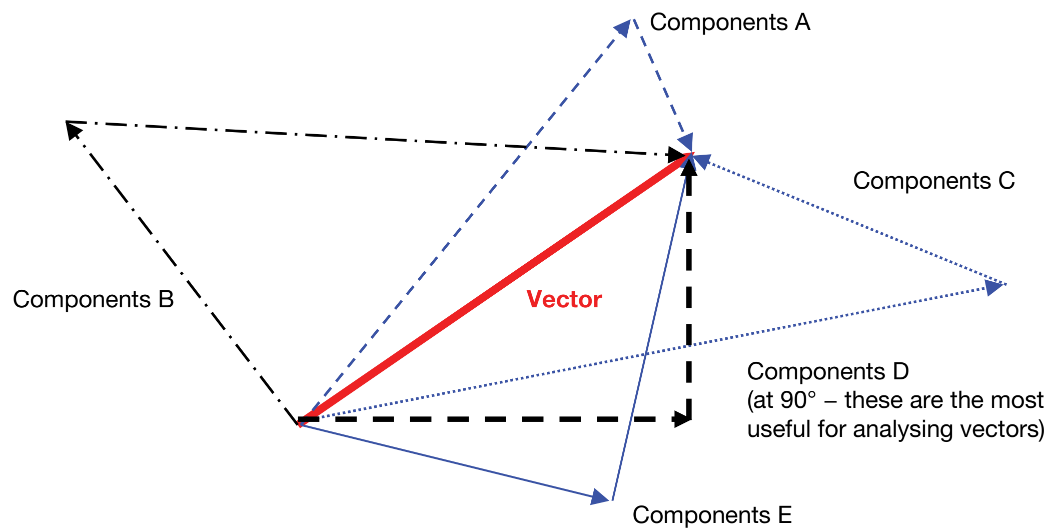
5.1 Components and Resolution Of Vectors

Analyse the motion of projectiles by resolving the motion into horizontal and vertical components, making the following assumptions: a constant vertical acceleration due to gravity and zero air resistance.

Components and resolution of vectors

The **components of a vector** are the **vectors we add together to get that vector**. For example, the vector shown below (in red) has many pairs of components (shown in various blues) and one pair at right angles (black).



When we refer to the components of a vector we specifically refer to the two vectors at 90° to each other, one **horizontal** and the other **vertical**, which would need to be added together to give that vector. In the diagram above, these would be components D, drawn in black.

When we **resolve** a vector into its components, then we are finding these two vectors at right angles.

Mathematically:

Horizontal component = vector cos θ

Vertical component = vector sin θ

Analysis of projectiles

- Horizontal and vertical components of projectile motion are independent of each other.
- Horizontal motion of a moving object is not subject to gravitational forces, and therefore experiences no acceleration.
- Vertical motion of an object near the surface of the Earth is affected by the downward force of gravity which gives it an acceleration of 9.8 ms^{-2}.

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Sample *Questions*

1. If the plane has a horizontal speed of 180 m s^{-1}, what is:

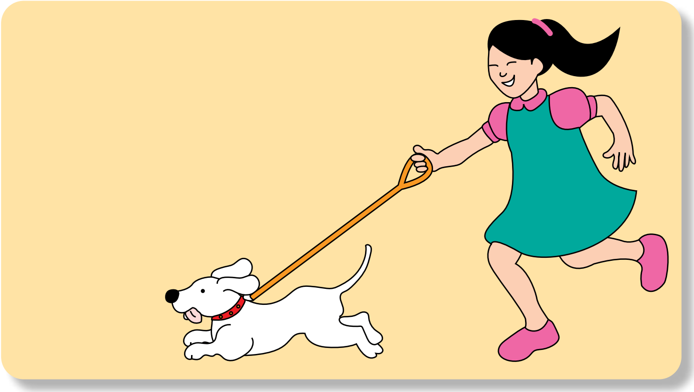
(a) It flight speed?

(b) Its vertical speed?

2. If the horizontal component of the tension in the leash held by the girl is 20 N what is:

(a) The tension in the leash?

(b) The vertical component of this tension?



3. If the vertical component of the tension in the leash is 18 N, find:

(a) The tension in the leash.

(b) The horizontal component on the tension.

4. If the missile in the photograph is moving at 2500 m s^{-1}, what are the components of its velocity?

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5.2 Analysing Projectile Motion

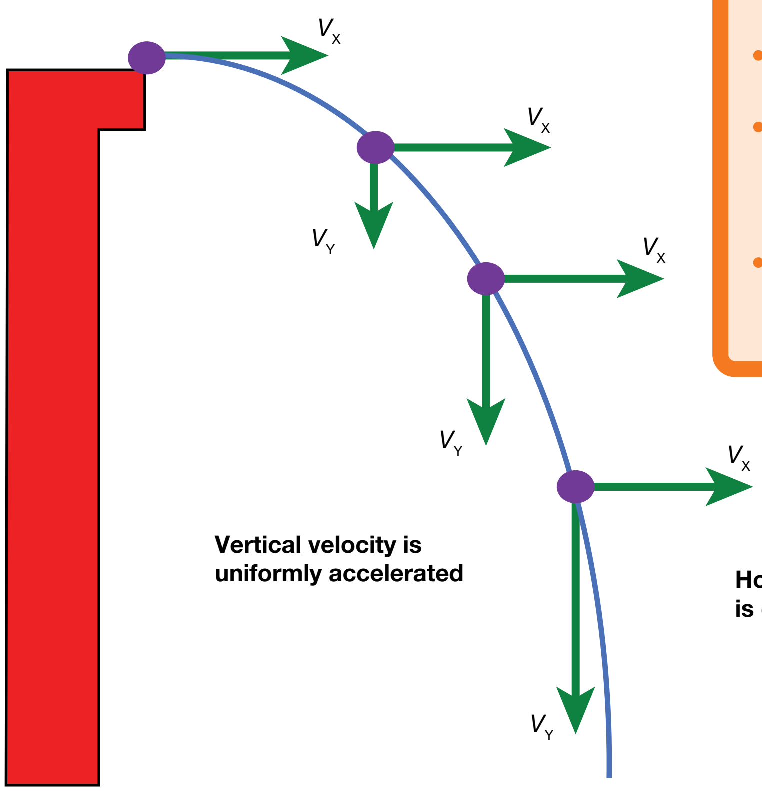
Apply the modelling of projectile motion to quantitatively derive the relationships between the following variables: initial velocity, launch angle, maximum height, time of flight, final velocity, launch height and horizontal range.

Analysing projectile motion

A projectile is any object thrown or shot into the air at any angle.

Trajectory of a projectile

- Only gravitational force acts on a projectile.
- The horizontal velocity of a projectile is constant.
- The vertical velocity of a projectile is accelerated – increasing if moving downwards, decreasing if moving upwards.
- The combination of these two velocities results in the object following a parabolic path.



Projectile motion and Newton's equations of motion

Equation used in straight line motion	Horizontal component of motion	Vertical component of motion
• $v = u + at$ • $v^2 = u^2 + 2as$ • $s = ut + \frac{1}{2}at^2$	• $u_x = u \cos \theta$ • $v_x = u_x$ $(a_x = 0)$ • $v_x^2 = u_x^2$ • $\Delta x = u_x t$	• $u_y = u \sin \theta$ • $v_y = u_y + a_y t$ • $v_y^2 = u_y^2 + 2a_y \Delta y$ • $\Delta y = u_y t + \frac{1}{2}a_y t^2$

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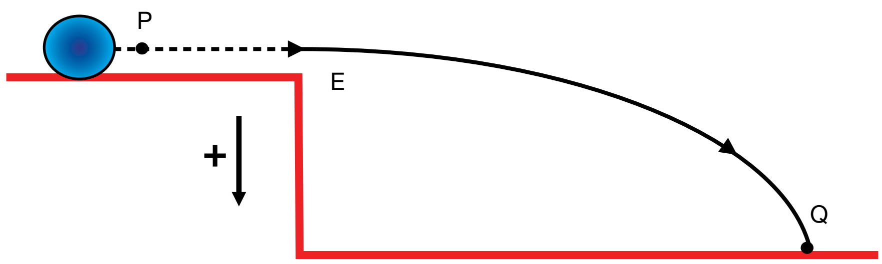
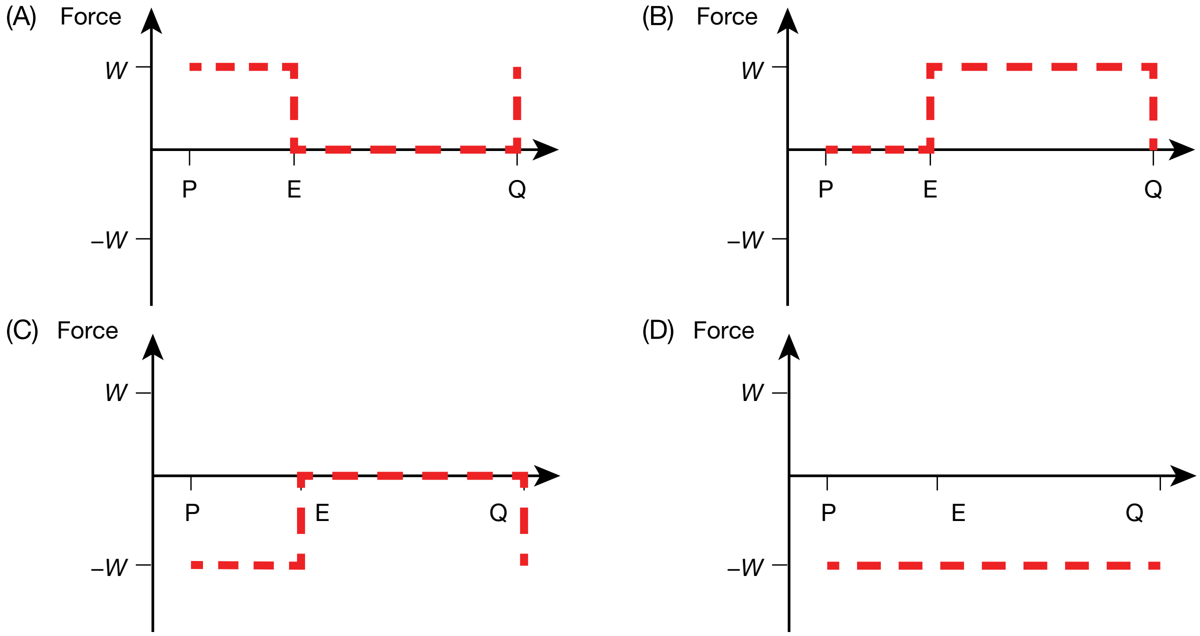
Sample Questions

1. A ball of weight W rolls across a horizontal surface and over the edge E, falling to the floor as shown in the diagram.

Which graph best shows the upward force acting on the ball as it moves from P to Q?





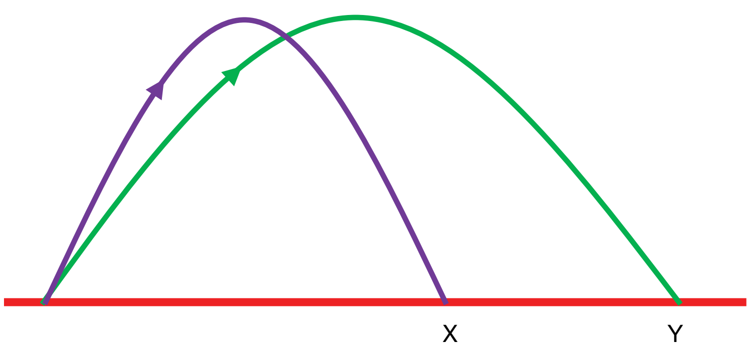
2. A projectile is fired from the top of a cliff with speed v at an angle θ above the horizontal.

Air resistance is negligible. What is the horizontal component of the projectile's velocity after time t?

(A) $v \cos \theta$ (B) $v \cos \theta - gt$ (C) $v \sin \theta - gt$ (D) $v \sin \theta$

3. The diagram shows the paths of two projectiles, X and Y, which rise to the same height. Which of the following is identical for both projectiles X and Y?

(A) Initial horizontal velocities.
(B) Initial vertical velocities.
(C) Initial velocities.
(D) Horizontal displacements.



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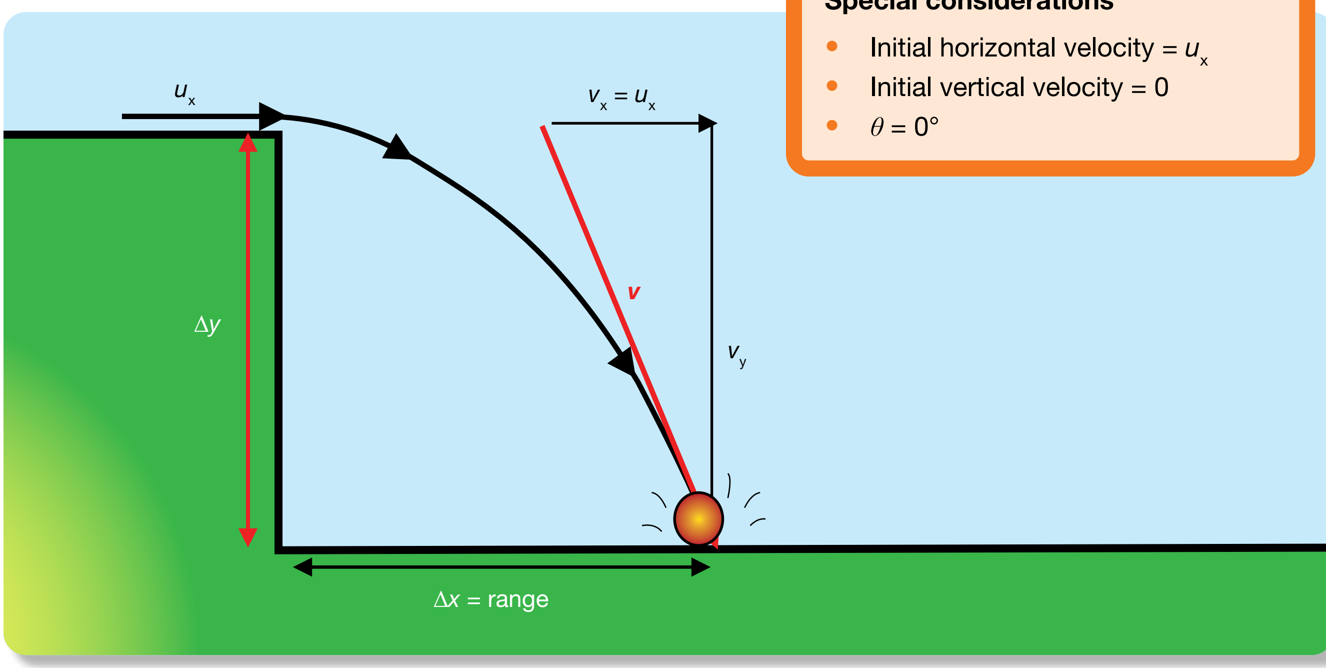
5.3 Projectile Motion Problems

Solve problems, create models and make quantitative predictions by applying the equations of motion relationships for uniformly accelerated and constant rectilinear motion.

Objects projected from a horizontal surface

Special considerations

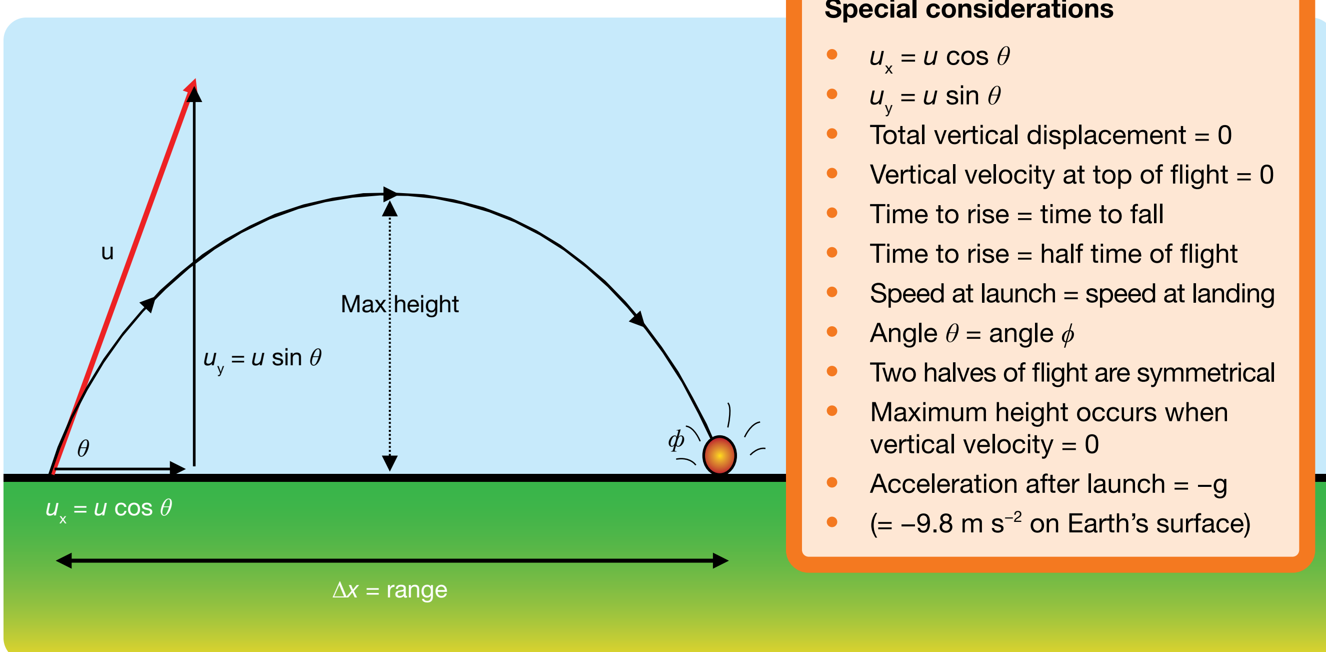
- Initial horizontal velocity = u_x
- Initial vertical velocity = 0
- $\theta = 0°$



Objects thrown up and landing at same level

Special considerations

- $u_x = u \cos \theta$
- $u_y = u \sin \theta$
- Total vertical displacement = 0
- Vertical velocity at top of flight = 0
- Time to rise = time to fall
- Time to rise = half time of flight
- Speed at launch = speed at landing
- Angle θ = angle ϕ
- Two halves of flight are symmetrical
- Maximum height occurs when vertical velocity = 0
- Acceleration after launch = –g
- (= –9.8 m s^{-2} on Earth's surface)



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Sample Questions

1. A tennis player, standing on the baseline, hits a tennis ball 1.5 m above the ground horizontally at 24 m s^{-1}. If the tennis court is 23.77 m long:

(a) What will be its time of flight?

(b) Will the ball land inside the court at the other end?

(c) What will be its velocity when it hits the ground?

2. A ball fired from a projectile gun at ground level just cleared 1.5 m high bar. The ball's horizontal velocity was 9.0 m s^{-1}.

(a) What was the ball's time of flight?

(b) How far out in front of the bar was the ball fired?

(c) What was its vertical launch velocity?

(d) At what angle was it fired?

3. An archer, standing on a raised platform, fires an arrow horizontally at 40 m s^{-1} at a height of 17.5 m above the ground at a tree which is 65 m away from her.

(a) What will be its time of flight?

(b) Where will it hit the tree?

(c) What is its velocity as it hits the tree?

4. A ball is launched at a velocity of 20 m s^{-1} at an angle of 25° to the horizontal.

(a) What was the ball's time of flight?

(b) What will be its maximum height above the launch position?

(c) What was its range?

5. A ball is launched at 40 m s^{-1} at an angle of 60° to the horizontal. What was its:

(a) Vertical launch velocity?

(b) Horizontal launch velocity?

(c) Time of flight?

(d) Range?

6. A 250 g toy truck moves off the edge of a table that is 1.25 m high and hits the floor 0.6 m out from the edge of the table.

(a) How long does it take to fall?

(b) At what speed did it leave the table?

(c) With what velocity did the truck hit the floor?

7. A ball thrown horizontally out from the top of a cliff at 40 m s^{-1} hits the ground 5 s later.

(a) What is the height of the cliff?

(b) What is the range of the ball?

(c) With what velocity does it hit the ground?

8. A ball kicked from ground level at an angle of 60° lands at the same level 10 s later.

(a) What is its initial horizontal velocity?

(b) What is its initial vertical velocity?

(c) What was its initial velocity?

(d) What was the ball's range?

(e) What was its maximum height?

9. A projectile is fired at 70 m s^{-1} at an angle of 53° to the ground. What is its:

(a) Time of flight?

(b) Range?

(c) Maximum height above the ground?

10. Police discover a car at the bottom of a 72 m cliff, 22 m out from the base of the cliff.

(a) How long did this car take to fall?

(b) At what speed did it go over the edge of the cliff?

(c) With what velocity did the car hit the ground at the bottom of the cliff?

11. A gun is fired horizontally toward a target 120 m away. It is aimed directly at the centre of the target. The bullet muzzle velocity is 200 m s^{-1}.

(a) What is its time of flight?

(b) How far below the centre of the target will the bullet hit?

12. A hawk in level flight 135 m above the ground accidentally drops the fish it caught. If the hawk's horizontal speed is 20.0 m s^{-1}:

(a) How long did the fish take to fall?

(b) How far ahead of where it was dropped will the fish hit the water?

13. In an experiment, two balls are rolled off a benchtop which is 1.25 m from the floor. Ball X, 2 kg, leaves the edge at 0.6 m s^{-1}. Ball Y, mass 3 kg, leaves the edge at 0.8 m s^{-1}.

(a) Which ball hits the floor first?

(b) How far apart do the balls hit the floor?

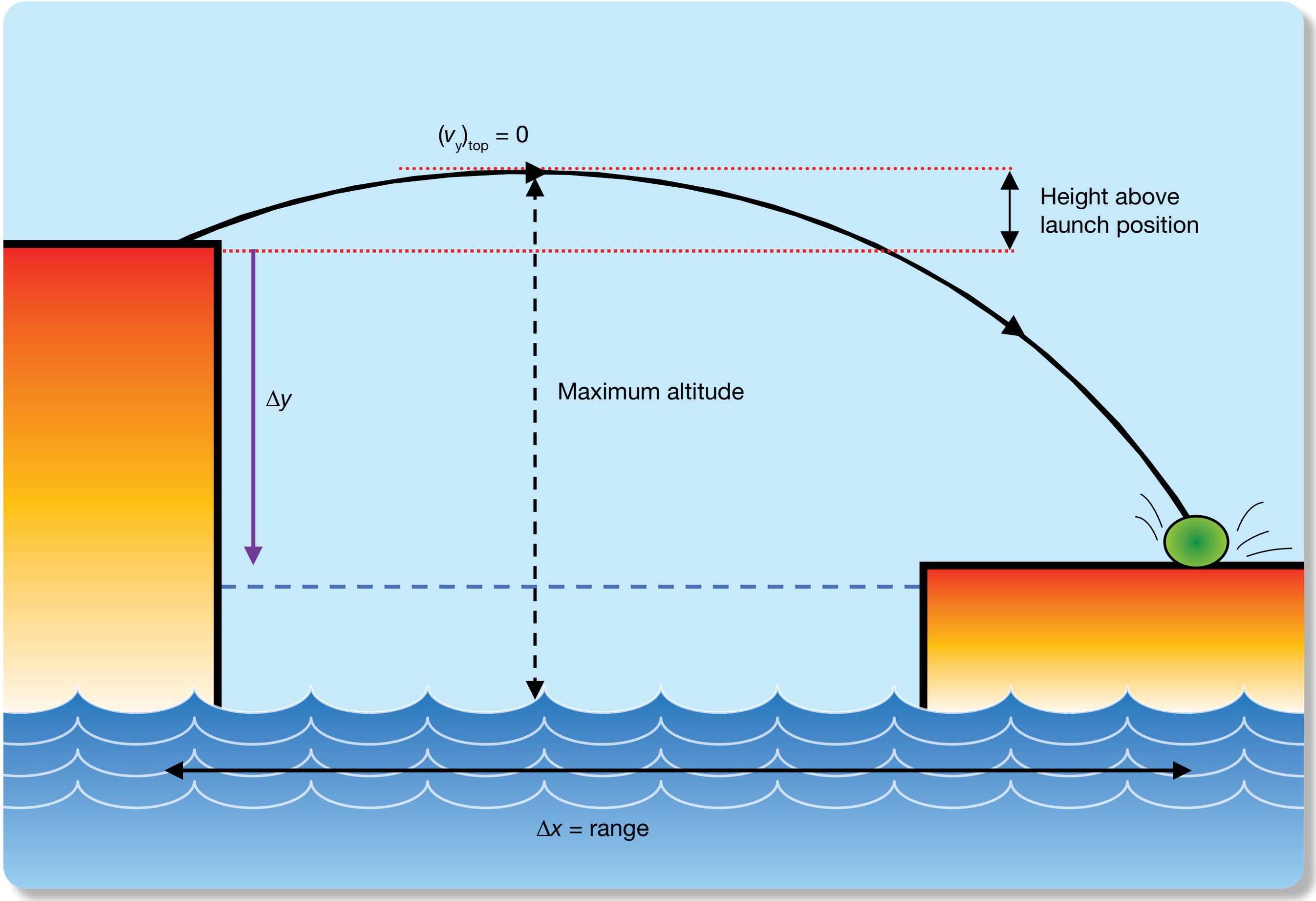
Objects landing at a different level

Note that for *all* projectiles, maximum range is achieved when launch angle = 45°.

Note also that the angle of projectiles with complementary launch angles will be the same.

Special considerations

- Vertical displacement = difference in height between the two levels
- If target lower (as shown), then vertical displacement is negative (assume upward direction positive)
- If target higher, vertical displacement positive
- Vertical velocity at top of flight = 0
- Time to rise **does not** equal time to fall
- Time to rise **is not** half time of flight
- Speed at launch **does not** equal speed at landing
- Angle of launch **does not** equal angle of landing
- Two halves of flight are **not** symmetrical
- Maximum height occurs when vertical velocity = 0
- Taking initial vertical velocity upwards as positive, **acceleration is negative**
- Range depends **only** on the horizontal component of the launch velocity
- Maximum height depends **only** on the vertical component of the launch velocity



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Sample *Questions*

1. A cannonball is fired from ground level and hits the wall of a castle 700 m away at a height of 146 m above the ground. It hits the wall when it is at its maximum altitude. Find:

(a) Its time of flight.

(b) Its initial horizontal velocity.

(c) Its initial vertical velocity.

(d) Its angle of inclination at launch.

(e) Its launch velocity.

(f) Its velocity as it hits the castle wall.

2. A boy runs straight off the end of a 12 m diving board with a speed of 2.2 m s^{-1}.

(a) How long before he hits the water?

(b) How far out from the end of the board will he hit the water?

(c) What will be his velocity when he hits the water?

3. A cannonball is fired at 35° to the ground from the top of a 60 m cliff at 25 m s^{-1}.

(a) What was its time of flight?

(b) Where will it hit the ground?

(c) At what velocity does it hit the ground?

(d) What will be its maximum height above the launch position?

4. A cannon is fired upwards at 150 m s^{-1} at 45°.

(a) What is its maximum height?

(b) What is its time of flight?

(c) What is its range?

(d) Where is it 9 seconds after firing?

(e) What is its velocity 9 s after firing?

5. A ball is thrown up from the top of a 64 m high building with an initial speed of 16 m s^{-1} at an angle of 10° from the vertical.

(a) What is its maximum height?

(b) What is its time of flight?

(c) What is its range?

(d) What will be its velocity on hitting the ground?

6. A projectile is fired into the air at 50° above the horizontal. It hits a target 900 m away on top of a 250 m high cliff 13 s later.

(a) What is its initial horizontal velocity?

(b) What was its initial velocity?

(c) What is its maximum height?

7. A projectile is fired into the air from the edge of a 125 m high cliff at an angle of 36.9° above the horizontal. It hits a target 455 m away from the base of the cliff 9.76 s later.

(a) What is its initial horizontal velocity?

(b) What is its initial vertical velocity?

(c) What is its initial velocity?

(d) What is its maximum height?

8. A projectile is fired into the air from the edge of a 400 m high cliff at an angle of 65° above the horizontal. It hits a target 2 km away from the base of the cliff 31 s later.

(a) What is its initial horizontal velocity?

(b) What is its initial vertical velocity?

(c) What is its launch speed?

(d) Find its maximum height.

9. A ball is kicked at 35° to the ground. It is 'headed' by a player 57.2 m away at a height of 1.8 m above the ground 2.79 s later.

(a) What was the ball's initial velocity?

(b) What is its horizontal speed when it hits the second player's head?

(c) What is its vertical speed when it hits the second player's head?

(d) What is its velocity when it hits the player's head?

10. A discus thrower releases the discus at a height of 1.2 m above the ground at 29.2 m s^{-1} at an angle of 45°. It lands after travelling 87.9 m.

(a) What is it time of flight?

(b) Find its maximum height above the ground.

(c) Find its velocity on hitting the ground.

11. A projectile is fired into the air at 40° above the horizontal. It hits a cliff 400 m above the launch point and 800 m away at the peak of its flight.

(a) Find its initial vertical velocity.

(b) Find its initial horizontal velocity.

(c) Find its launch velocity.

(d) How long does it take to reach the building?

(e) Find its velocity as it passes over the building.

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Answers

5.1 Components and Resolution Of Vectors

1. (a) Flight speed = 202.2 m s^{-1} (measuring θ as 27°)
(b) Vertical speed = 91.8 m s^{-1}
2. (a) Tension = 24.4 N (measuring θ as 35°)
(b) Vertical component = 14.0 N
3. (a) Tension = 29.2 N (measuring θ as 38°)
(b) Horizontal component = 23.0 N
4. (a) Vertical component = 1135 m s^{-1} (measuring θ as 27°)
(b) Horizontal component = 2227.5 m s^{-1}

5.2 Analysing Projectile Motion

1. C
2. A
3. B

5.3 Projectile Motion Problems

Objects projected from a horizontal surface

Objects thrown up and landing at same level

Note: Your answers may differ slightly due to rounding off errors.

1. (a) 0.55 s
(b) Yes – it will be 10.5 m inside the back line
(c) 24.6 m s^{-1} at 12.7° down from horizontal
2. (a) 1.11 s
(b) 5.0 m
(c) 5.42 m s^{-1}
(d) 31.1° up from horizontal
3. (a) 1.625 s (note that it hits the tree before it reaches the ground, so 1.9 s is incorrect)
(b) 12.94 m below its release point (4.56 m above the ground)
(c) 43.05 m s^{-1} at 21.7° down from horizontal
4. (a) 1.72 s
(b) 3.64 m
(c) 31.2 m
5. (a) 34.6 m s^{-1}
(b) 20 m s^{-1}
(c) 7.1 s
(d) 141.2 m
6. (a) 0.51 s
(b) 1.19 m s^{-1}
(c) 5.1 m s^{-1} at 14° to the vertical
7. (a) 122.5 m
(b) 200 m
(c) 63.25 m s^{-1} at 50.8° down from horizontal
8. (a) 28.3 m s^{-1}
(b) 49 m s^{-1}
(c) 56.58 m s^{-1} at 60° up from horizontal
(d) 283 m
(e) 122.5 m
9. (a) 11.4 s
(b) 480.6 m
(c) 159.5 m
10. (a) 3.83 s
(b) 5.74 m s^{-1}
(c) 38 m s^{-1} at 81.3° down from horizontal
11. (a) 0.6 s
(b) 1.76 m
12. (a) 5.25 s
(b) 105 m
13. (a) They will both hit the floor at the same time (0.505 s)
(b) Ball Y lands 0.1 m further out than ball X (0.4 m compared to 0.3 m)

Objects landing at a different level

1. (a) 5.46 s
(b) 128.2 m s^{-1}
(c) 53.5 m s^{-1}
(d) 22.7° up from horizontal
(e) 138.9 m s^{-1} at 22.7° up from horizontal
(f) 128.2 m s^{-1} horizontally (it is at the top of its flight)
2. (a) 1.56 s
(b) 3.43 m
(c) 15.4 m s^{-1} at 8.2° to the vertical
3. (a) 5.25 s
(b) 107.6 m out from the base of the cliff
(c) 42.5 m s^{-1} at 28.9° to the vertical
(d) 10.5 m
4. (a) 574 m
(b) 21.65 s
(c) 2295.9 m
(d) 557.7 m above the ground and 954 m out horizontally from the launch point
(e) 107.5 m s^{-1} at 9.5° up from the horizontal
5. (a) 12.7 m
(b) 5.56 s
(c) 15.44 m
(d) 38.9 m s^{-1} down at 4.1° to the vertical
6. (a) 69.2 m s^{-1}
(b) 107.7 m s^{-1}
(c) 347.3 m

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